Worldline Action for All-Spin Compton Scattering

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New Trends in QFT,

Amplitudes and Gravity,

Cd. Universitaria Puebla, Mexico

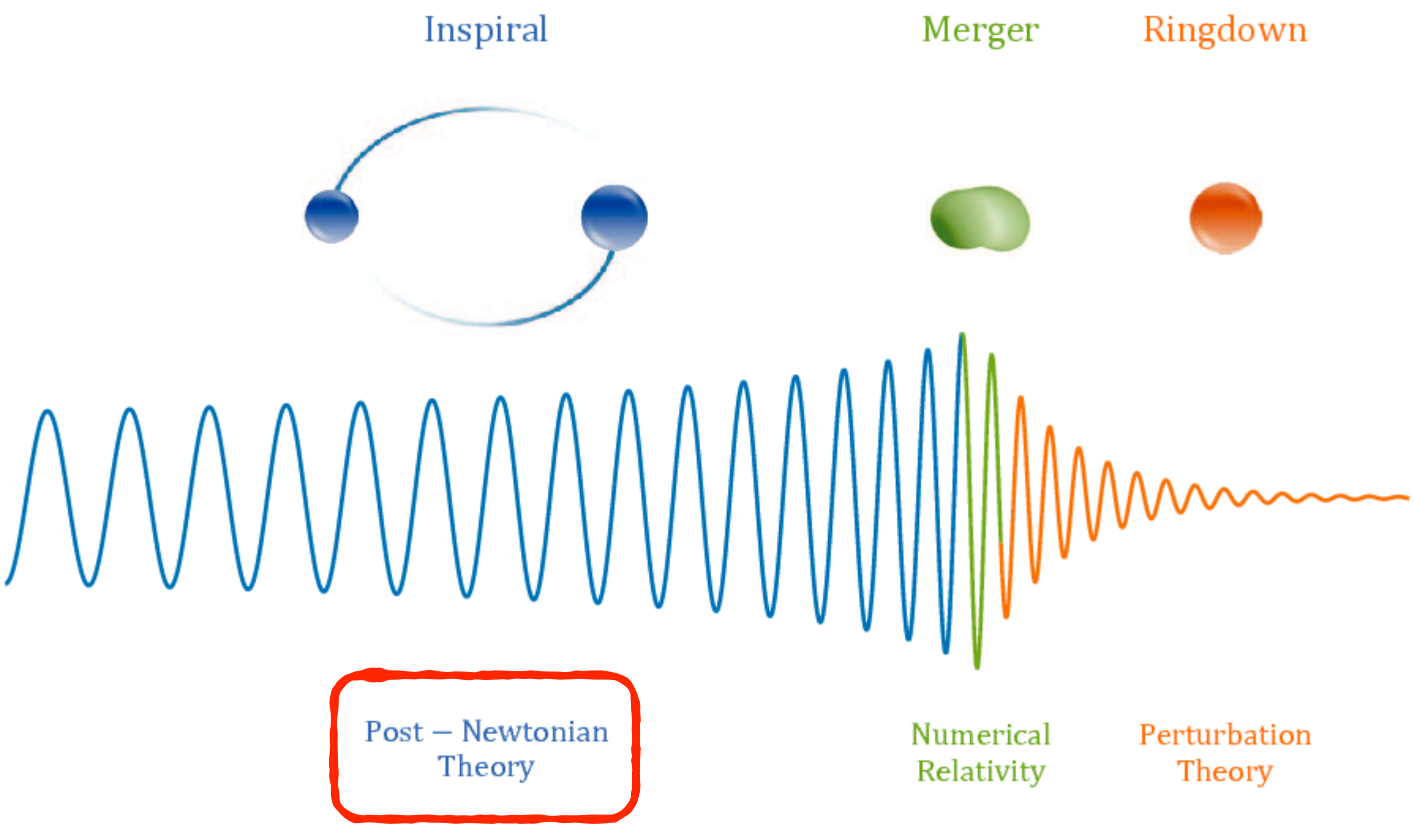
Based on refs:

Cangemi, Chiodaroli, HJ, Ochirov, Pichini, Skvortsov

[2212.06120], [2312.14913], [2212.06120]

Ben-Shahar, Cangemi, HJ [2512.24549]

Motivation: gravitational waves



Inspiral phase: high experimental sentitivity ←→ need theoretical precision

→ errors accumulate

→ analytic control possible

→ important for LISA band

PN, PM and spin-multipole expansions

Post-Newtonian (PN) expansion:

expand in G and v

Bound systems:

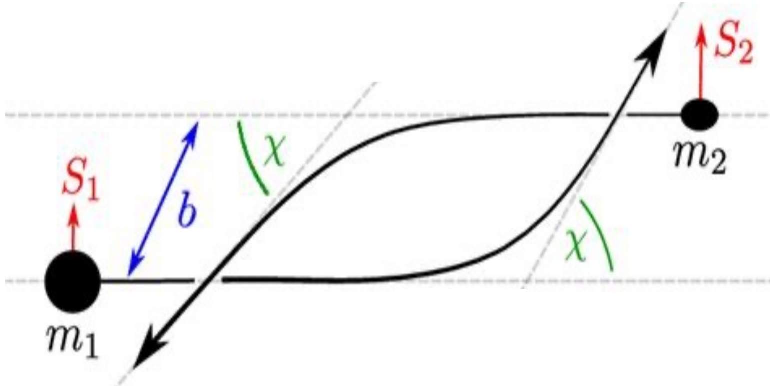
$$v^2 \sim \frac{GM}{r}$$

(virial theorem)

Post-Minkowskian (PM) expansion:

Gravitational scattering:

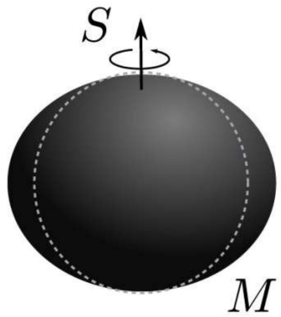
expand in G → loop expansion



Spin-multipole expansion:

Rotating black holes

expand in S_1 and S_2



<u>Methods</u>
- BH perturbation theory
- Worldine EFTs
- Heavy-mass EFTs
- Quantum scattering ampl's
- Higher-spin QFTs

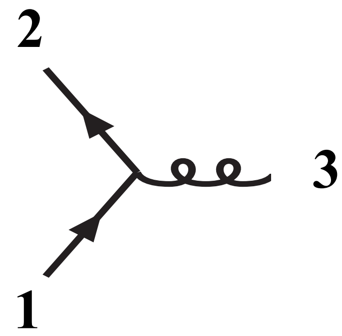
Outline

- AHH amplitudes and low-spin Kerr theories
- Higher-spin construction for Kerr
- Worldline Compton computation to all spin orders
- Discussion

AHH and Low-spin Kerr

Kerr 3pt interactions

Natural higher-spin gravitational 3pt amplitudes:



$$M(1\phi^s, 2\bar{\phi}^s, 3h^+) = im^2x^2\frac{\langle\mathbf{12}\rangle^{2s}}{m^{2s}},$$

$$M(1\phi^s, 2\bar{\phi}^s, 3h^-) = i\frac{m^2}{x^2}\frac{[\mathbf{12}]^{2s}}{m^{2s}}$$

Arkani-Hamed, Huang, Huang ('17)

Linearized energy-momentum tensor for Kerr source

Vines ('17)

$$T^{\mu\nu}(-k) = 2\pi\,\delta(p\cdot k)\,p^{(\mu}\exp(m^{-1}S*ik)^{\nu)}{}_{\rho}\,p^{\rho}$$

Non-minimal worldline action for Kerr:

Levi, Steinhoff ('15)

$$L_{\rm SI} = \sum_{n=1}^{\infty}\frac{(-1)^n}{(2n)!}\frac{C_{ES^{2n}}}{m^{2n-1}}D_{\mu_{2n}}\cdots D_{\mu_3}\frac{E_{\mu_1\mu_2}}{\sqrt{u^2}}S^{\mu_1}S^{\mu_2}\cdots S^{\mu_{2n-1}}S^{\mu_{2n}}$$
$$+\sum_{n=1}^{\infty}\frac{(-1)^n}{(2n+1)!}\frac{C_{BS^{2n+1}}}{m^{2n}}D_{\mu_{2n+1}}\cdots D_{\mu_3}\frac{B_{\mu_1\mu_2}}{\sqrt{u^2}}S^{\mu_1}S^{\mu_2}\cdots S^{\mu_{2n-1}}S^{\mu_{2n}}S^{\mu_{2n+1}}$$

spin-multipole Wilson coeff: $C_{ES^{2n}} = C_{BS^{2n+1}} = 1$ (Kerr BH)

root-Kerr gauge theory

Classical double copy → Kerr-Schild form

Monteiro, O'Connell ('14)

metric: $g_{\mu\nu} = \eta_{\mu\nu} + \phi k_\mu k_\nu$ (Kerr, double copy)

gauge field: $A_\mu = \phi k_\mu$ (root-Kerr, single copy)

$$k^\mu k_\mu = 0 \qquad \phi(r) = \frac{2MGr^3}{r^4 + a^2 z^2} \qquad \frac{x^2+y^2}{r^2+a^2} + \frac{z^2}{r^2} = 1$$

Newman-Janis shift:

$$\Psi^{\text{Kerr}}(x) = \Psi^{\text{Schwarzschild}}(x + ia)$$

$$\Phi^{\sqrt{\text{Kerr}}}(x) = \Phi^{\text{Coulomb}}(x + ia)$$

(Newman-Penrose curvature scalars)

→ classical 3pt amplitudes

$$M_{3,\pm}^{\text{Kerr}} = e^{\pm p\cdot a} M_{3,\pm}^{\text{Schwarzchild}}$$

$$A_{3,\pm}^{\sqrt{\text{Kerr}}} = e^{\pm p\cdot a} A_{3,\pm}^{\text{Coulomb}}$$

Guevara, Ochirov, Vines;
Arkani-Hamed, Huang, O'Connell;
Guevara, Maybee, Ochirov, O'Connell, Vines

AHH amplitudes → Kerr BH ?

Arkani-Hamed, Huang, Huang. (‘17)

Spin-s gravitational 3pt amplitudes:

$$M(1\phi^s, 2\bar{\phi}^s, 3h^+) = im^2x^2\frac{\langle\mathbf{12}\rangle^{2s}}{m^{2s}}\,, \qquad M(1\phi^s, 2\bar{\phi}^s, 3h^-) = i\frac{m^2}{x^2}\frac{[\mathbf{12}]^{2s}}{m^{2s}}$$

Spin-s gauge theory 3pt amplitudes

$$A(1\phi^s, 2\bar{\phi}^s, 3A^+) = mx\frac{\langle\mathbf{12}\rangle^{2s}}{m^{2s}}\,, \qquad A(1\phi^s, 2\bar{\phi}^s, 3A^-) = \frac{m}{x}\frac{[\mathbf{12}]^{2s}}{m^{2s}}$$

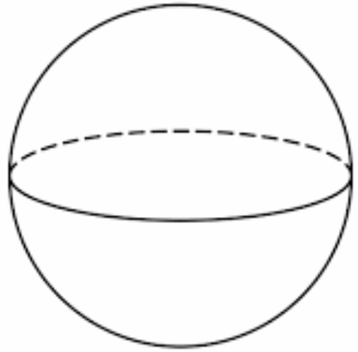
Q1: Where is the spin vector ? $S^\mu = ma^\mu$

Q2: Where is the exponential factor ? $e^{\pm p\cdot a}$

Q3: What are the quantum theories ? (before classical limit)

Quantum spin operator

Introduce projective 3-sphere coordinates



$$z^a = (x_1 + i x_2, x_3 + i x_4) \quad \to \quad 1 = z^a \bar{z}_a = |x|^2$$

parametrizes SU(2) <-> spin $\qquad z^a \sim (|\uparrow\rangle, |\downarrow\rangle)$

Relation between classical spin vector and quantum spin:

$$S^\mu = \frac{s}{2m} (\bar{z}^a z_a)^{2s-1} \left(\langle \bar{\mathbf{1}} | \sigma^\mu | \mathbf{1}] + \langle \mathbf{1} | \sigma^\mu | \bar{\mathbf{1}}] \right)$$

massive spinor-helicity formalism

Properties:

Transversality of spin vector: $\quad p_1 \cdot S = 0$

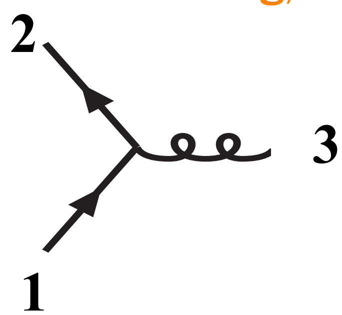
Equals an expectation value: $\quad S^\mu = \langle \hat{S}^\mu \rangle \equiv (\bar{z})^{2s} \cdot \hat{S}^\mu \cdot (z)^{2s}$

Gives spin operator: $\quad [\hat{S}^\mu, \hat{S}^\nu] = i \epsilon^{\mu\nu\rho} \hat{S}_\rho \qquad \hat{S}^2 = s(s+1) \mathbb{1}$

AHH amplitudes = Kerr BHs

Guevara, Ochirov, Vines;
Chung, Huang, Kim, Lee ('18)

Relate in/out states by Lorentz transf.



$$|\mathbf{2}\rangle := |\bar{\mathbf{1}}\rangle + p_3 \cdot \sigma |\bar{\mathbf{1}}]/(2m).$$

AHH factor → exponential of spin operator:

$$\frac{\langle \mathbf{12} \rangle^{2s}}{m^{2s}} = \Big\langle \sum_{n=0}^{2s} \frac{1}{n!} \Big(\frac{p_3 \cdot \hat{S}}{m} \Big)^n \Big\rangle = \langle e^{p_3 \cdot \hat{a}} \rangle$$

Quantum Kerr and root-Kerr 3pt → Quantum Newman-Janis shift

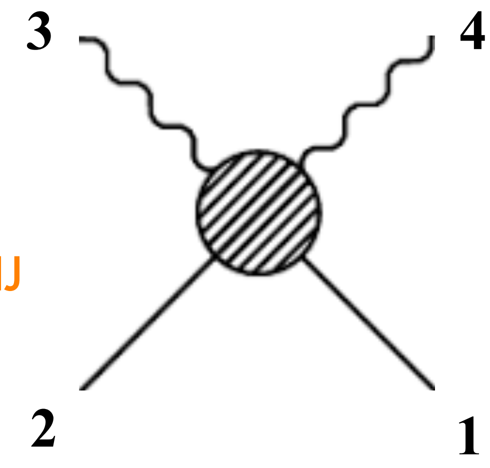
$$M_{3,\pm}^{\text{Kerr}} = \langle e^{\pm p_3 \cdot \hat{a}} \rangle M_{3,\pm}^{\text{Schwarzchild}}$$

$$A_{3,\pm}^{\sqrt{\text{Kerr}}} = \langle e^{\pm p_3 \cdot \hat{a}} \rangle A_{3,\pm}^{\text{Coulomb}}$$

with ring-radius (spin) operator: $\hat{a}^\mu = \frac{\hat{S}^\mu}{m}$

Kerr Compton amplitudes

Candidate Compton amplitudes via BCFW:



same helicity case:

$$M(1\phi^s, 2\bar{\phi}^s, 3h^+, 4h^+) = i\frac{\langle \mathbf{12}\rangle^{2s}[34]^4}{m^{2s-4}s_{12}t_{13}t_{14}}$$

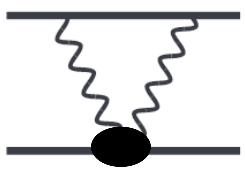
Ochirov, HJ

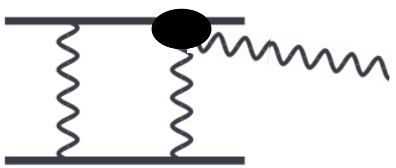
oppsite helicity case:

$$M(1\phi^s, 2\bar{\phi}^s, 3h^-, 4h^+) = i\frac{[4|p_1|3\rangle^{4-2s}([4\mathbf{1}]\langle 3\mathbf{2}\rangle + [4\mathbf{2}]\langle 3\mathbf{1}\rangle)^{2s}}{s_{12}t_{13}t_{14}}$$

AHH

Needed for NLO calculations:





Spurious pole for $s > 2$ → contact term completion (not unique)

$$\frac{1}{[4|p_1|3\rangle^{2s-4}}$$

Kerr and root-Kerr = fundamental theories ?

Elementary particle Lagrangians behind Kerr at $s \leq 2$ — Arkani-Hamed, Huang, O'Connell

EFTs	$s = {}^1\!/_2$	$s = 1$	$s = {}^3\!/_2$	$s = 2$	$s = {}^5\!/_2$	$s \geq 3$
Kerr	Major.	Proca	Rar.-Sch.	KK grav.	SHP	HS
$\sqrt{\text{Kerr}}$	Dirac	W-boson	gravitino	HS	HS	HS

Cangemi, Chiodaroli, HJ, Ochirov, Pichini, Skvortsov

[HS=higher-spin;SHP=Singh-Hagen;Porrati]

Lagrangians worked out in [2107.14779] Chiodaroli, HJ, Pichini

At most one non-minmal Riemann interaction at $s = 2,\ 5/2$

$$\Phi^s \Phi^s R_{\mu\nu\rho\sigma}$$

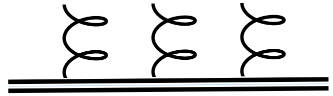
Double copy works up to $s = 2$ Ochirov, HJ

$$M_n^{\text{Kerr}} \sim \left[A_n^{\sqrt{\text{Kerr}}}(\mathbf{1}^{s/2}, \mathbf{2}^{s/2}, 3, 4, \ldots, n)\right]^2$$

All elementary-particle Lagrangians = massless theory + Kaluza-Klein!

Low-spin Compton double copies

Kerr amplitudes for $s \leq 2$ admit Compton double copy (also *n*-points)



$$(\text{YM} + \text{scalar}) \otimes (\text{YM} + \text{scalar}) = (\text{GR} + \text{scalar})$$

$$(\text{YM} + \text{scalar}) \otimes (\text{YM} + \text{fermion}) = (\text{GR} + \text{fermion})$$

Ochirov, HJ

$$(\text{YM} + \text{scalar}) \otimes (\text{YM} + \text{W-boson}) = (\text{GR} + \text{Proca})$$

$$(\text{YM} + \text{W-boson}) \otimes (\text{YM} + \text{fermion}) = (\text{GR} + \text{massive gravitino})$$

$$(\text{YM} + \text{W-boson}) \otimes (\text{YM} + \text{W-boson}) = (\text{GR} + \text{massive KK graviton})$$

Lagrangians unique: no new interaction terms beyond cubic order

Chiodaroli, HJ, Pichini

Can be used for $(S^\mu)^{\leq 4}$ PM/PN calculations [Spin Universality holds!]

2-to-2: $G^3 S^2$ Akpinar, Febres Cordero, Kraus, Smirnov, Zeng

Compton: $G^2 S^4$ Akpinar

Waveform: $G^3 S^2$ Bohnenblust, Ita, Kraus, Schlenk

Higher-Spin (HS) Construction

Higher-spin Kerr theory principles

1) Higer-spin massive gauge symmetry Stueckelberg; Zinoviev

→ Correct DOF and good high-energy behavior

$$\Delta(\epsilon,\bar\epsilon)=\sum_{s=0}^{\infty}(\epsilon)^s\cdot\Delta^{(s)}\cdot(\bar\epsilon)^s=\underbrace{\frac{1}{p^2-m^2+i0}\,\frac{1-\frac{1}{4}\epsilon^2\bar\epsilon^2}{1+\epsilon\cdot\bar\epsilon+\frac{1}{4}\epsilon^2\bar\epsilon^2}}_{\text{Feynman gauge}}$$

Cangemi, Chiodaroli, HJ, Ochirov, Pichini, Skvortsov

2) Chiral higher-spin formalism
→ off-shell spinor-helicity
→ Lagrangians as simple as amplitudes

Ochirov, Skvortsov; Gordon Chalmers; CCJOPS

3) Contact interactions from Homogeneous Symmetric Polynomials

→ Generalizes geometic sums/series
→ Classical limit gives exponentials

$$P_n^{(2s)}(\varsigma_1,\ldots,\varsigma_n)$$

CCJOPS

Using HS gauge invariance

Cangemi, Chiodaroli, HJ, Ochirov, Pichini, Skvortsov

Consider spin-2 root-Kerr case:

physical field: $\Phi_{\mu\nu}$

Stückelberg fields: $\{B_\mu, \varphi\}$

Imposing a linearized massive higher-spin gauge transformation:

$$\delta\Phi_{\mu\nu} = \frac{1}{2}\partial_\mu\xi_\nu + \frac{1}{2}\partial_\nu\xi_\mu + \frac{m}{\sqrt{2}}\eta_{\mu\nu}\xi,$$
$$\delta B_\mu = \partial_\mu\xi + \frac{m}{\sqrt{2}}\xi_\mu,$$
$$\delta\varphi = \sqrt{3}m\xi,$$

← gauge parameter

Makes sure that:

→ DOFs are correct,

→ small-mass limit better behaved than naively expected

Massive Ward identities

We write down ansatz for off-shell interactions:

Cangemi, Chiodaroli, HJ, Ochirov, Pichini, Skvortsov

$$V_{\Phi\overline{\Phi}A} \sim m\,(\epsilon_1)^2\,(\epsilon_2)^2\,\epsilon_3\left(\frac{p^3}{m^3}+\frac{p}{m}\right),$$

$$V_{B\overline{\Phi}A} \sim m\,(\epsilon_1)\,(\epsilon_2)^2\,\epsilon_3\left(\frac{p^2}{m^2}+1\right),$$

$$V_{\varphi\overline{\Phi}A} \sim m\,(\epsilon_2)^2\,\epsilon_3\left(\frac{p}{m}\right),$$

and constrain them using Ward identities

$$V_{\xi\overline{\Phi}A}\big|_{(2,3)} = V_{\zeta\overline{\Phi}A}\big|_{(2,3)} = 0$$

where the vertices corresponding to gauge parameters are:

$$V_{\xi\overline{\Phi}A} := \frac{m}{\sqrt{2}}V_{B\overline{\Phi}A} - \frac{i}{2}p_1\cdot\frac{\partial}{\partial\epsilon_1}V_{\Phi\overline{\Phi}A},$$

$$V_{\zeta\overline{\Phi}A} := \sqrt{3}mV_{\varphi\overline{\Phi}A} - ip_1\cdot\frac{\partial}{\partial\epsilon_1}V_{B\overline{\Phi}A} + \frac{m}{2\sqrt{2}}\left(\frac{\partial}{\partial\epsilon_1}\right)^2 V_{\Phi\overline{\Phi}A}.$$

→ 3pt amplitude: $A(\Phi_1^2\,\overline{\Phi}_2^2\,A_3^+) = A_0\frac{\langle\mathbf{12}\rangle^3}{m^4}(c_1[\mathbf{12}] + (1-c_1)\langle\mathbf{12}\rangle)$

unique after current constraint: $c_1 = 0$

General spin-s EFTs

Consider tower $k = 0, 1, 2, \ldots, s$ of HS fields and gauge parameters:

$$\Phi^k := \Phi^{\mu_1 \mu_2 \cdots \mu_k}, \qquad \xi^k := \xi^{\mu_1 \mu_2 \cdots \mu_k}$$

(double-traceless) (traceless)

Zinoviev (2001)

Gauge transformation: $$\delta \Phi^k = \partial^{(1} \xi^{k-1)} + m \alpha_k \xi^k + m \beta_k \eta^{(2} \xi^{k-2)}$$

$$\alpha_k = \frac{1}{k+1} \sqrt{\frac{(s-k)(s+k+1)}{2}}, \qquad \beta_k = \frac{1}{2} \frac{k}{k-1} \alpha_{k-1}$$

Minimal Lagrangian: $$\mathcal{L}_0 = \mathcal{L}_{\rm F} + \frac{1}{2} \sum_{k=0}^{s-1} (-1)^k (k+1) G^k G^k$$

Gauge-fixing fn: $$G^k = \partial \cdot \Phi^{k+1} - \frac{k}{2} \partial^{(1} \tilde{\Phi}^{k+1)} + m \big(\alpha_k \Phi^k - \gamma_k \tilde{\Phi}^{k+2} - \delta_k \eta^{(2} \tilde{\Phi}^{k)} \big)$$

Feynman-gauge Lagr: $$\mathcal{L}_{\rm F} = \sum_{k=0}^{s} \frac{(-1)^k}{2} \Big[\Phi^k (\Box + m^2) \Phi^k - \frac{k(k-1)}{4} \tilde{\Phi}^k (\Box + m^2) \tilde{\Phi}^k \Big]$$

Cangemi, Chiodaroli, HJ,
Ochirov, Pichini, Skvortsov

Chiral higher-spin approach

Ochirov, Skvortsov

Easier way to get correct DOFs:

Change Lorentz rep. $$(s,s) \longrightarrow (2s,0)$$

Chiral fields $$|\Phi\rangle := \Phi_{\alpha_1\ldots\alpha_{2s}}$$ $SL(2,\mathbb{C})$ indices

Minimal Lagrangian $$\mathcal{L}^{(s)}_{\text{min.}} = \langle D_\mu\Phi|D^\mu\Phi\rangle - m^2\langle\Phi|\Phi\rangle$$

Gives correct all-plus helicity (AHH; OJ) amplitudes:

$$A_n(1^s,2^s,3^+,4^+,\ldots,n^+) = \langle\mathbf{12}\rangle^{2s}A_n^{\text{scalar}}$$

However, breaks parity, and also naive renormalizability…

W-bosons in SM: $$\mathcal{L}^{(1)} = \langle\Phi|\left\{|\overleftarrow{D}|\overrightarrow{D}|\otimes\frac{1}{1-\frac{ig}{m^2}|F_-|}\right\}|\Phi\rangle - m^2\langle\Phi|\Phi\rangle + \mathcal{O}(\Phi^4)$$

Chalmers, Siegel

Universal polynomials

Complete homogenous symmetric polynomials: Cangemi, Chiodaroli, HJ, Ochirov, Pichini, Skvortsov

$$P_n^{(k)} = \frac{\varsigma_1^k}{(\varsigma_1 - \varsigma_2)(\varsigma_1 - \varsigma_3)\dots(\varsigma_1 - \varsigma_n)} + \text{perm}(\varsigma_1, \varsigma_2, \dots \varsigma_n)$$

variables: $\varsigma_1 = \langle \mathbf{1}|1{+}4|\mathbf{2}]\,,\quad \varsigma_2 = \langle \mathbf{2}|2{+}3|\mathbf{1}]\,,\quad \varsigma_3 = m\langle \mathbf{21}\rangle\,,\quad \varsigma_4 = m[\mathbf{21}]$

Classical limit gives entire functions: $\varsigma_i \to x, y, z\,;\qquad \varsigma_i^{2s} \to e^x, e^y, e^z$

Conjecture: (contact) interactions linear in $P_n^{(k)}$: $C^{(s)} = C^{(s)}[P_n^{(k)}]$

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<u>Conjecture</u>: (contact) interactions linear in $P_n^{(k)}$: $C^{(s)} = C^{(s)}[P_n^{(k)}]$

Chiral higher-spin version of Levi-Steinhoff: CCJOPS

$$\mathcal{L}_{\mathrm{Kerr}} = \sqrt{-g}\Big\{\frac{1}{2}\langle\nabla_\mu\Phi|\nabla^\mu\Phi\rangle - \frac{m^2}{2}\langle\Phi|\Phi\rangle - \frac{1}{4}\underbrace{\sum_{k=0}^{2s-2}\frac{2s-k-1}{m^{2k}}\langle\Phi|\Big\{\big(|\overleftarrow{\nabla}|\overrightarrow{\nabla}|\big)^{\odot k}}_{P_3^{(2s)}(1,\,|\overleftarrow{\nabla}|\overrightarrow{\nabla}|/m^2,\,1)}\odot|R_-|\Big\}|\Phi\rangle\Big\}$$

Non-minimal chiral interactions

Restore parity at 3pts → AHH 3pt amplitudes:

Cangemi, Chiodaroli, HJ, Ochirov, Pichini, Skvortsov

Root-Kerr non-minimal interactions:

$$\mathcal{L}^{(s)} = \langle D_\mu \Phi | D^\mu \Phi \rangle - m^2 \langle \Phi | \Phi \rangle + \sum_{k=0}^{2s-1} \frac{ig}{m^{2k}} \langle \Phi | \Big\{ |\overleftarrow{D}|\overrightarrow{D}|^{\odot k} \otimes |F_-| \Big\} |\Phi\rangle + \mathcal{O}(F^2)$$

Opposite-helicity 4pt amplitude modified (still parity inv.)

$$A(\mathbf{1}^s, \mathbf{2}^s, 3^-, 4^+)$$

$$= \frac{\langle 3|1|4]^2 (U+V)^{2s}}{m^{4s} t_{13} t_{14}} - \frac{\langle \mathbf{13}\rangle \langle 3|1|4] [4\mathbf{2}]}{m^{4s} t_{13}} P_2^{(2s)} + \frac{\langle \mathbf{13}\rangle \langle 3\mathbf{2}\rangle [\mathbf{1}4][4\mathbf{2}]}{m^{4s}} P_2^{(2s-1)}$$

$$- \frac{\langle \mathbf{13}\rangle \langle 3\mathbf{2}\rangle [\mathbf{1}4][4\mathbf{2}]}{m^{4s-2}} \langle \mathbf{12}\rangle [\mathbf{12}] P_4^{(2s-1)} + C^{(s)} \, .$$

↑ universal polynomials

Constraints for fixing contact term

- well-behaved classical limit $s \to \infty$;
- amplitudes with $s < 2$ not modified: $C^{(s<2)} = 0$;
- compatible with massive higher-spin gauge invariance;
- s-independent numerical coefficients;
- parity invariance
- all contact terms have spinor-helicity structure $\sim \langle \mathbf{1}3\rangle\langle 3\mathbf{2}\rangle[\mathbf{1}4][4\mathbf{2}]$;
- classical spin quadrupole fixed by $s = 1$ amplitude;
- no dissipation effects, nor contributions from non-perturbative considerations.

→ classical amplitude fixed
→ but quantum amplitudes only partially fixed

Candidate Kerr Compton ampl.

$$M(\mathbf{1}^s,\mathbf{2}^s,3^-,4^+) = \frac{\langle 3|1|4]^4 P_1^{(2s)}}{m^{4s} s_{12} t_{13} t_{14}} - \frac{\langle \mathbf{1}3\rangle[4\mathbf{2}]\langle 3|1|4]^3}{m^{4s} s_{12} t_{13}} P_2^{(2s)} + \frac{\langle \mathbf{1}3\rangle\langle 3\mathbf{2}\rangle[\mathbf{1}4][4\mathbf{2}]}{m^{4s} s_{12}} \left(\langle 3|1|4]^2 P_2^{(2s-1)} + m^4 \langle 3|\rho|4]^2 P_4^{(2s-1)}\right)$$
$$+ \frac{\langle \mathbf{1}3\rangle\langle 3\mathbf{2}\rangle[\mathbf{1}4][4\mathbf{2}]}{m^{4s-2} s_{12}} \langle 3|1|4]\langle 3|\rho|4]\left(P_2^{(2s-2)} - m^2 \langle \mathbf{12}\rangle[\mathbf{12}] P_4^{(2s-2)}\right)$$
$$+ \frac{\langle \mathbf{1}3\rangle^2\langle 3\mathbf{2}\rangle^2[\mathbf{1}4]^2[4\mathbf{2}]^2}{2m^{4s-4}} \langle \mathbf{12}\rangle[\mathbf{12}]\left[(1+\eta)P_{5|\varsigma_1}^{(2s-2)} + (1-\eta)P_{5|\varsigma_2}^{(2s-2)}\right] + \alpha\, C_\alpha^{(s)}.$$

→ Classical Kerr amplitude: Cangemi, Chiodaroli, HJ, Ochirov, Pichini, Skvortsov

$$\mathcal{M}(\mathbf{1},\mathbf{2},3^-,4^+) = \frac{(p\cdot\chi)^4}{q^2\,(p\cdot q_\perp)^2}\left(e^x \cosh z - w\, e^x \operatorname{sinhc} z + \frac{w^2-z^2}{2} E(x,y,z)\right)$$
$$+ \frac{(p\cdot\chi)^4}{q^2\,(p\cdot q_\perp)^2} \frac{w^2-z^2}{2} (w-x)\tilde{E}(x,y,z)$$
$$- \frac{(p\cdot\chi)^4}{(p\cdot q_\perp)^4} \frac{(w^2-z^2)^2}{2}\left(\frac{\partial \tilde{E}}{\partial x} + \eta \frac{\partial \tilde{E}}{\partial z}\right) \qquad + \alpha z\,(\text{polygamma terms})$$

Candidate Kerr Compton ampl.

$$M(\mathbf{1}^s,\mathbf{2}^s,3^-,4^+) = \frac{\langle 3|1|4]^4 P_1^{(2s)}}{m^{4s} s_{12} t_{13} t_{14}} - \frac{\langle \mathbf{13}\rangle[\mathbf{42}]\langle 3|1|4]^3}{m^{4s} s_{12} t_{13}} P_2^{(2s)} + \frac{\langle \mathbf{13}\rangle\langle \mathbf{32}\rangle[\mathbf{14}][\mathbf{42}]}{m^{4s} s_{12}} \left(\langle 3|1|4]^2 P_2^{(2s-1)} + m^4 \langle 3|\rho|4]^2 P_4^{(2s-1)}\right)$$
$$+ \frac{\langle \mathbf{13}\rangle\langle \mathbf{32}\rangle[\mathbf{14}][\mathbf{42}]}{m^{4s-2} s_{12}} \langle 3|1|4]\langle 3|\rho|4]\left(P_2^{(2s-2)} - m^2 \langle \mathbf{12}\rangle[\mathbf{12}] P_4^{(2s-2)}\right)$$
$$+ \frac{\langle \mathbf{13}\rangle^2\langle \mathbf{32}\rangle^2[\mathbf{14}]^2[\mathbf{42}]^2}{2m^{4s-4}} \langle \mathbf{12}\rangle[\mathbf{12}]\left[(1+\eta)P_{5|\varsigma_1}^{(2s-2)} + (1-\eta)P_{5|\varsigma_2}^{(2s-2)}\right] + \alpha\, C_\alpha^{(s)}.$$

→ **Classical Kerr amplitude:** Cangemi, Chiodaroli, HJ, Ochirov, Pichini, Skvortsov

$$\mathcal{M}(\mathbf{1},\mathbf{2},3^-,4^+) = \frac{(p\cdot\chi)^4}{q^2\,(p\cdot q_\perp)^2}\left(e^x \cosh z - w\, e^x \operatorname{sinhc} z + \frac{w^2-z^2}{2} E(x,y,z)\right)$$
$$+ \frac{(p\cdot\chi)^4}{q^2\,(p\cdot q_\perp)^2} \frac{w^2-z^2}{2} (w-x)\tilde{E}(x,y,z)$$
$$- \frac{(p\cdot\chi)^4}{(p\cdot q_\perp)^4} \frac{(w^2-z^2)^2}{2} \left(\frac{\partial \tilde{E}}{\partial x} + \eta \frac{\partial \tilde{E}}{\partial z}\right) \qquad + \alpha z\,(\text{polygamma terms})$$

$$x = a\cdot(q_4 - q_3)\,, \quad y = a\cdot q$$
$$z = 2|a|\omega\,, \quad w = 2\omega\langle 3|a|4]/\langle 3|v|4]$$

$$E(x,y,z) = \frac{e^y - e^x \cosh z + (x-y)e^x \operatorname{sinhc} z}{(x-y)^2 - z^2} + (y \to -y)$$

→ Matches Teukolsky calc. (far-zone) Bautista, Guevara, Kavanagh, Vines up to S^7 $(\alpha = 0)$

→ Compatible with Bautista, Bonelli, Iossa, Tanzini, Zhou up to S^8; also 2PM calc. Bohnenblust, Cangemi, HJ, Pichini

Worldline Compton to all spins

Kerr worldline action (linear in *R*)

See [2311.01430] for proper gauge fixing & WQFT Feynman rules

Spinning particle worldline:

$$S=-\int d\tau\Big(p_\mu\dot{x}^\mu+\frac{1}{2}S_{\mu\nu}\Omega^{\mu\nu}-\frac{\ell}{2}(p^2-\mathcal{M}^2)-\frac{1}{|p|}\frac{Dp^\mu}{d\tau}S_{\mu\nu}\Lambda_0^\nu\Big)$$

Dynamical mass function: $\mathcal{M}^2=m^2+\mathcal{L}_R+\mathcal{L}_{R^2}+\mathcal{O}(R^3)$

Levi-Steinhoff non-mimimal curvature (*R*) terms for Kerr:

$$\mathcal{L}_R=\frac{2m^2}{(S\cdot\nabla)^2}\Big[\Big(1-e^{\frac{i}{m}S\cdot\nabla}+\frac{i}{m}S\cdot\nabla\Big)E^+_{SS}+\Big(1-e^{-\frac{i}{m}S\cdot\nabla}-\frac{i}{m}S\cdot\nabla\Big)E^-_{SS}\Big]$$

(anti-)self-dual curvature: $E^\pm_{\mu\nu}:=\frac{1}{2}(E_{\mu\nu}\pm iB_{\mu\nu})$

Kerr worldline action (linear in *R*)

See [2311.01430] for proper gauge fixing & WQFT Feynman rules

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$$S = -\int d\tau \Big(p_\mu \dot{x}^\mu + \frac{1}{2} S_{\mu\nu}\Omega^{\mu\nu} - \frac{\ell}{2}(p^2 - \mathcal{M}^2) - \frac{1}{|p|}\frac{Dp^\mu}{d\tau} S_{\mu\nu}\Lambda_0^\nu \Big)$$

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Levi-Steinhoff non-mimimal curvature (*R*) terms for Kerr:

$$\mathcal{L}_R = \frac{2m^2}{(S\cdot\nabla)^2}\Big[\Big(1 - e^{\frac{i}{m}S\cdot\nabla} + \frac{i}{m}S\cdot\nabla\Big)E^+_{SS} + \Big(1 - e^{-\frac{i}{m}S\cdot\nabla} - \frac{i}{m}S\cdot\nabla\Big)E^-_{SS}\Big]$$

(anti-)self-dual curvature: $E^{\pm}_{\mu\nu} := \frac{1}{2}(E_{\mu\nu} \pm iB_{\mu\nu})$

Polar scattering kinematics: Bautista, Guevara, Kavanagh, Vines

→ non-minimal terms vanish to linear order:

$$S^\nu E^{\pm}_{\mu\nu} = E^{\pm}_{\mu S} = 0$$

Ben-Shahar, Cangemi, HJ

→ Compton ampliude has no contact terms (e.g. BCFW works)

WQFT approach → WL Feynman rules

Ben-Shahar

Expand WL fields around flat background:

$$p_\mu \to m v_\mu + \pi_\mu \,,$$
$$x^\mu \to b^\mu + v^\mu \tau + z^\mu \,,$$
$$S_{\mu\nu} \to S_{\mu\nu} + s_{\mu\nu} \,,$$
$$\Lambda_I^\mu \to \Lambda_I^\mu + \lambda^{\mu\nu}\Lambda_{I\nu} + \frac{1}{2}\lambda^{\mu\nu}\lambda_{\nu\rho}\Lambda_I^\rho + \ldots$$

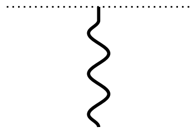
Invert quadratic part of action:

$$\langle z^\mu(-\omega) z^\nu(\omega)\rangle = -\, i\frac{1}{m\omega^2}\eta^{\mu\nu} - \frac{1}{m^2\omega}S^{\mu\nu} \,,$$
$$\langle \pi^\mu(-\omega) z^\nu(\omega)\rangle = -\,\frac{1}{\omega}\eta^{\mu\nu} \,,$$
$$\langle s_{\mu\nu}(-\omega) s_{\rho\sigma}(\omega)\rangle = -\,\frac{2}{\omega}(\eta_{\nu[\sigma}S_{\rho]\mu} - \eta_{\mu[\sigma}S_{\rho]\nu}) \,,$$
$$\langle s_{\mu\nu}(-\omega) \lambda_{\rho\sigma}(\omega)\rangle = \frac{2}{\omega}\eta_{\mu[\rho}\eta_{\sigma]\nu} \,,$$
$$\langle \lambda^{\mu\nu}(-\omega) z_\rho(\omega)\rangle = -\,\frac{2}{m\omega}v^{[\mu}\delta_\rho^{\nu]} \,.$$

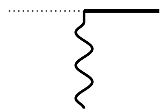
Expand metric on WL:

$$h_{\mu\nu}(x) = \int \frac{d^4k}{(2\pi)^4} e^{ik\cdot(b+v\tau+z)} h_{\mu\nu}(k)$$

Feynman vertex rules:



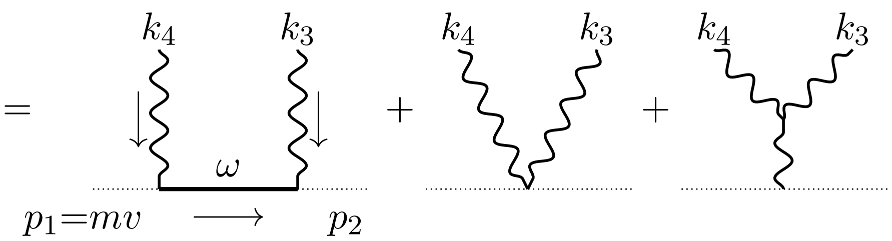
$$= -\frac{i}{2}m(v\cdot h\cdot v) - \frac{1}{2}(v\cdot h\cdot S\cdot k) - i\langle \mathcal{L}_R\rangle\Big|_h$$



$$= \frac{m}{2}(v\cdot h\cdot v)(k\cdot z) - i(\pi\cdot h\cdot v) - \frac{i}{2}(v\cdot h\cdot S\cdot k)(z\cdot k) - \frac{1}{2}(v\cdot h\cdot s\cdot k)$$
$$- \frac{i}{2}(z\cdot h\cdot S\cdot k)\omega + \frac{1}{2}(v\cdot h\cdot v)(v\cdot\lambda\cdot S\cdot k) - \frac{1}{2}(v\cdot h\cdot v)(v\cdot s\cdot k) - i\langle \mathcal{L}_R\rangle\Big|_{h,W}$$

Compute Compton:

$$M(1,2,3^\pm,4^\pm) =$$



Compton from linear-in-R worldline

Ben-Shahar, Cangemi, HJ

Same-helicity ansatz (manifest polar-scattering vanishing):

$$M(1,2,3^+,4^+) = m^2 \frac{[34]^4 e^y}{(v\cdot q_\perp)^2 q^2} + m^2 [34]^4 \frac{u^2 - z^2}{2(v\cdot q_\perp)^4} \sum_{k=1}^{3} u^{k-1} z^{3-k} f_k^{++}(x,y,z)$$

Opposite-helicity ansatz (manifest polar-scattering vanishing):

$$M(1,2,3^-,4^+) = m^2 \frac{\langle 3|v|4]^4 E_{\text{pole}}}{(v\cdot q_\perp)^2 q^2} + m^2 \langle 3|v|4]^4 \frac{w^2 - z^2}{2(v\cdot q_\perp)^4} \sum_{k=1}^{3} w^{k-1} z^{3-k} f_k^{-+}(x,y,z)$$

Entire function for pole residue (spurious-pole free):

$$E_{\text{pole}} := \frac{1}{2}\left\{ e^x(1-w) + w^2 \frac{y+x-w}{y}\left[\frac{e^y - e^x}{(x-y)^2} + \frac{e^x}{x-y}\right]\right\} = e^{x-w} + (\text{contacts})$$

Classical variables:

$$x = a\cdot(q_4 - q_3)\,, \quad y = a\cdot q$$
$$z = 2|a|\omega\,, \quad w = 2\omega\langle 3|a|4]/\langle 3|v|4]$$
$$u = 2\omega[3|va|4]/[34]$$

Polar scattering kinematics:

$$w = \pm z = u$$

Worldline computation re-summed

Opposite-helicity WL entire functions:

After computing up to S^{16} we can extrapolate & resum entire functions:

Ben-Shahar, Cangemi, HJ

$$f_3^{-+} := \frac{2e^x x + 2x + x^2 - y^2 - (4x + x^2 - y^2)e^{\frac{x}{2}} \cosh\frac{y}{2}}{(x^2-y^2)^2}\,,$$

$$f_2^{-+} := \frac{8e^x x - (2+2e^x+x)(8+x^2-y^2) - \frac{1}{4}(x^2-y^2)^2}{(x^2-y^2)^3} + e^{\frac{x}{2}}\frac{(32+x^3-xy^2)\cosh\frac{y}{2} - y(16+x^2-y^2)\sinh\frac{y}{2}}{(x^2-y^2)^3}\,,$$

$$f_1^{-+} := \frac{8x(2x+x^2-y^2)e^{\frac{x}{2}}\cosh\frac{y}{2}}{(x^2-y^2)^3} + \frac{4y(4x-x^2+y^2)e^{\frac{x}{2}}\sinh\frac{y}{2}}{(x^2-y^2)^3} + \frac{e^y}{2y^2(x-y)^2} + \frac{e^{-y}}{2y^2(x+y)^2} - \frac{x^2+3y^2}{y^2(x^2-y^2)^2} - \frac{3}{2}\frac{x^2-y^2+4x}{(x^2-y^2)^2} - \frac{16x^2e^x}{(x^2-y^2)^3} + \frac{2e^x(x+1)}{(x^2-y^2)^2}\,,$$

Same-helicity WL entire functions:

$$f_3^{++} := \frac{8x(2y+y^2-x^2)e^{\frac{y}{2}}\sinh\frac{x}{2}}{(y^2-x^2)^3} + \frac{4(4x^2+y(x^2-y^2))e^{\frac{y}{2}}\cosh\frac{x}{2}}{(y^2-x^2)^3} + \frac{x^2+x^2y+5y^3}{y^2(x^2-y^2)^2} - \frac{3}{2}\frac{x^2-y^2-2}{(x^2-y^2)^2} + \frac{e^y(17y^4-x^4)}{y^2(x^2-y^2)^3} + \frac{2e^y(y+7)}{(x^2-y^2)^2}$$

$$f_1^{++} := -\, f_3^{-+}\Big|_{x\leftrightarrow y}\,,$$

$$f_2^{++} := \left(f_2^{-+} - \frac{1}{4}f_3^{-+} + \frac{1}{2}f_0^{-+}\right)\Big|_{x\leftrightarrow y}\,,$$

Worldline computation re-summed

Opposite-helicity WL entire functions:

$$f_3^{-+} := \frac{2e^x x + 2x + x^2 - y^2 - (4x + x^2 - y^2)e^{\frac{x}{2}} \cosh \frac{y}{2}}{(x^2 - y^2)^2} ,$$

$$f_2^{-+} := \frac{8e^x x - (2 + 2e^x + x)(8 + x^2 - y^2) - \frac{1}{4}(x^2 - y^2)^2}{(x^2 - y^2)^3} + e^{\frac{x}{2}} \frac{(32 + x^3 - xy^2) \cosh \frac{y}{2} - y(16 + x^2 - y^2) \sinh \frac{y}{2}}{(x^2 - y^2)^3} ,$$

$$f_1^{-+} := \frac{8x(2x + x^2 - y^2)e^{\frac{x}{2}} \cosh \frac{y}{2}}{(x^2 - y^2)^3} + \frac{4y(4x - x^2 + y^2)e^{\frac{x}{2}} \sinh \frac{y}{2}}{(x^2 - y^2)^3} + \frac{e^y}{2y^2(x - y)^2} + \frac{e^{-y}}{2y^2(x + y)^2} - \frac{x^2 + 3y^2}{y^2(x^2 - y^2)^2} - \frac{3}{2}\frac{x^2 - y^2 + 4x}{(x^2 - y^2)^2} - \frac{16x^2 e^x}{(x^2 - y^2)^3} + \frac{2e^x(x + 1)}{(x^2 - y^2)^2} ,$$

After computing up to S^{16} we can extrapolate & resum entire functions:

Ben-Shahar, Cangemi, HJ

Same-helicity WL entire functions:

$$f_3^{++} := \frac{8x(2y + y^2 - x^2)e^{\frac{y}{2}} \sinh \frac{x}{2}}{(y^2 - x^2)^3} + \frac{4(4x^2 + y(x^2 - y^2))e^{\frac{y}{2}} \cosh \frac{x}{2}}{(y^2 - x^2)^3} + \frac{x^2 + x^2 y + 5y^3}{y^2(x^2 - y^2)^2} - \frac{3}{2}\frac{x^2 - y^2 - 2}{(x^2 - y^2)^2} + \frac{e^y(17y^4 - x^4)}{y^2(x^2 - y^2)^3} + \frac{2e^y(y + 7)}{(x^2 - y^2)^2}$$

$$f_1^{++} := -f_3^{-+}\Big|_{x\leftrightarrow y} ,$$

$$f_2^{++} := \left(f_2^{-+} - \frac{1}{4}f_3^{-+} + \frac{1}{2}f_0^{-+}\right)\Big|_{x\leftrightarrow y} ,$$

This is infinite garbage! Same-helicity Kerr should not have contact terms!

[BGKV Teukolsky cal.]

Comparing with HS Compton

Opposite-helicity WL entire functions:

$$f_3^{-+} := \frac{2e^x x + 2x + x^2 - y^2 - (4x + x^2 - y^2)e^{\frac{x}{2}} \cosh\frac{y}{2}}{(x^2 - y^2)^2} ,$$

$$f_2^{-+} := \frac{8e^x x - (2 + 2e^x + x)(8 + x^2 - y^2) - \frac{1}{4}(x^2 - y^2)^2}{(x^2 - y^2)^3}$$
$$+ e^{\frac{x}{2}} \frac{(32 + x^3 - xy^2)\cosh\frac{y}{2} - y(16 + x^2 - y^2)\sinh\frac{y}{2}}{(x^2 - y^2)^3} ,$$

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$$- \frac{x^2 + 3y^2}{y^2(x^2 - y^2)^2} - \frac{3}{2}\frac{x^2 - y^2 + 4x}{(x^2 - y^2)^2} - \frac{16x^2 e^x}{(x^2 - y^2)^3} + \frac{2e^x(x + 1)}{(x^2 - y^2)^2} ,$$

Opposite-helicity entire functions from higher-spin QFT:

CCJOPS

$$E(x, y, z) := \frac{e^y - e^x \cosh z + (x - y)e^x \operatorname{sinhc} z}{(x - y)^2 - z^2} + (y \to -y)$$

$$\tilde{E}(x, y, z) := \frac{1}{2y} \frac{e^y - e^x \cosh z + (x - y)e^x \operatorname{sinhc} z}{(x - y)^2 - z^2} + (y \to -y)$$

Analytic structure very different!

Comparing with HS Compton

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$$f_3^{-+} := \frac{2e^x x + 2x + x^2 - y^2 - (4x + x^2 - y^2)e^{\frac{x}{2}} \cosh\frac{y}{2}}{(x^2 - y^2)^2},$$

$$f_2^{-+} := \frac{8e^x x - (2 + 2e^x + x)(8 + x^2 - y^2) - \frac{1}{4}(x^2 - y^2)^2}{(x^2 - y^2)^3} + e^{\frac{x}{2}} \frac{(32 + x^3 - xy^2)\cosh\frac{y}{2} - y(16 + x^2 - y^2)\sinh\frac{y}{2}}{(x^2 - y^2)^3},$$

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Also infinite garbage!

No easy matching to Teukolsky results.

Opposite-helicity entire functions from higher-spin QFT:

CCJOPS

$$E(x, y, z) := \frac{e^y - e^x \cosh z + (x - y)e^x \operatorname{sinhc} z}{(x - y)^2 - z^2} + (y \to -y)$$

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Analytic structure very different!

Corrections: add Kerr worldline R^2 terms

Ben-Shahar, Cangemi, HJ

New quadratic curvature corrections:

6 entire functions

$$\mathcal{L}_{R^2} = E^-_{S\mu} O^{\mu\nu}_{-+} E^+_{S\nu} + E^+_{S\mu} O^{\mu\nu}_{++} E^+_{S\nu} \; + \; \text{h.c.}$$

$$O^{\mu\nu}_{\pm\pm} := \left(S^\mu S^\nu - \frac{1}{2} g^{\mu\nu} S^2\right) \mathcal{F}_1^{\pm\pm} - \frac{1}{2} g^{\mu\nu} S^2 \mathcal{F}_2^{\pm\pm} - \frac{i}{m} S^2 \mathcal{F}_3^{\pm\pm} \left(S^\nu \nabla^\mu - g^{\mu\nu} S \cdot \nabla\right)$$

Simplest operator-valued entire function:

$$\mathcal{F}_1^{++} = i \frac{m(\overleftarrow{\nabla} \cdot S + S \cdot \nabla)(e^{-\overleftarrow{\nabla} \cdot S \frac{i}{m}} - 1)(e^{-\frac{i}{m} S \cdot \nabla} - 1) + (\overleftarrow{\nabla} \cdot S)(S \cdot \nabla)(e^{-\overleftarrow{\nabla} \cdot S \frac{i}{m}} + e^{-\frac{i}{m} S \cdot \nabla} - 2)}{8 m^4 (\overleftarrow{\nabla} \cdot S)^2 (S \cdot \nabla)^2}$$

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New quadratic curvature corrections:

6 entire functions

$$\mathcal{L}_{R^2} = E^{-}_{S\mu} O^{\mu\nu}_{-+} E^{+}_{S\nu} + E^{+}_{S\mu} O^{\mu\nu}_{++} E^{+}_{S\nu} \; + \; \text{h.c.}$$

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Q & A: 1) Do these R^2 operators vanish in static limit? No!

2) Are they tidal/Love numbers? No! Far-zone terms!

3) What Kerr property is captured? None! (WL garbage)

4) Could near-zone (loop) matching be a cure? Likely not!

If Casimir $S^2=0$ → FZ-NZ amiguity absent?, but garbage remains

5) Any cure in sight? chiral/self-dual worldline ←→ chiral QFT

Conclusions: Kerr dynamics from HS vs. WL

- Higher-spin Kerr theory works remarkable well (e.g. AHH 3pt)
- Low-spin Lagrangians correspond to elementary particles
- HS gauge symmetry, chiral fields, and universal polynomials → Classical Compton with good agreement vs Teukolsky eq.
- Using spinning worldline with linear-in-R interactions works well for spin $S^{\leq 4}$ but at higher orders→ unwanted terms!
- Resummed the WL Compton amplitude into entire functions, compared to HS Compton→ difference given as R^2 operators
- The R^2 operators likely do not describe anything physical, instead they remove unwanted WL contributions.

Kerr dynamics is a challenging but highly rewarding problem!


Gracias!