Memory effect with NUT charge

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Motivation: use scattering amplitudes to predict GW effects from exotic objects

Outline: Introduction to NUT charge, memory
KMOC formalism
Nutty amplitudes
Soft theorems
Nutty memory

1 Introduction

1.1 NUT charge

Taub-NUT solution (50-60's) : m, n

$$ds^2 = -f(r)\left(dt + 2n\cos\theta\, d\phi\right)^2 + \frac{dr^2}{f(r)} + (r^2+n^2)\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

$$f(r) = \frac{r^2 - 2mr - n^2}{}$$

$r^2 + n^2$

- large r : $g_{t\phi} = -2n\cos\theta$ $\rightarrow$ gravitomagnetic monopole
  similar to $A = -g\cos\theta\, d\phi$ $\rightarrow$ magnetic monopole

- exact map : classical double copy

EM	Gravity
(e, g)	(m, n)
$A_\mu = e\,\phi\, k_\mu + g\,\psi\, l_\mu$	$g_{\mu\nu} = \eta_{\mu\nu} + m\,\phi\, k_\mu k_\nu + n\,\psi\, l_\mu l_\nu$
Dirac string	Misner string
	Closed timelike curves !
	Prediction : geodesics ✓
	waveform ?

1.2 Memory effect

$$\Delta \zeta_A = \frac{\mathcal{E}_{AB}(\hat{x})}{4\pi r}\, \zeta^B$$

- memory tensor $\mathcal{E}_{AB}(\hat{x})$ $\leftrightarrow$ Weinberg soft theorem (see later)

- measurable by LISA, etc
- effect NUT charge ?

② KMOC formalism

In-state $|\psi\rangle$ Out-state $S|\psi\rangle$

$$S = \mathbb{1} + iT \qquad S^\dagger S = \mathbb{1}$$

Observable $\mathcal{O}$:

$$\Delta\mathcal{O} = \langle\psi|S^\dagger \mathcal{O} S|\psi\rangle - \langle\psi|\mathcal{O}|\psi\rangle =$$

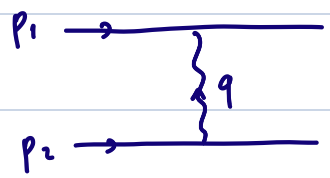
$$= i\langle\psi|[\mathcal{O},T]|\psi\rangle + \langle\psi|T^\dagger[\mathcal{O},T]|\psi\rangle$$

Classical limit $\hbar\to 0$ (modes: $k_\mu = \hbar\bar{k}_\mu$, $\epsilon = \hbar\omega$)

- Impulse Δp_1^μ at leading order (large separation):

$$\Delta p_1^\mu = \lim_{\hbar\to 0} i\int \hat{d}^4 q\, \hat{\delta}(2p_1\cdot q)\,\hat{\delta}(2p_2\cdot q)\, e^{-\frac{i}{\hbar} b\cdot q}\, q^\mu A_4$$

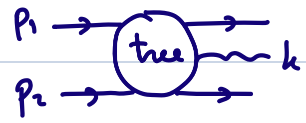
- impact parameter b_μ, $b = \sqrt{|b_\mu b^\mu|}$

- $A_4 =$ 

$$= \frac{1}{q^2}\sum_{\eta=\pm} A_3[p_1, q^{(\eta)}]\, A_3[p_2,(-q)^{(-\eta)}]$$

$$+ \underbrace{\mathcal{O}((q^2)^0)}_{\rightsquigarrow 0 \text{ at } b>0}$$

- Waveform $R_{\mu\nu\lambda\rho}(x)$: leading order from

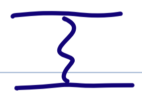
$A_5 =$ 

③ Dyonic / Nutty amplitudes

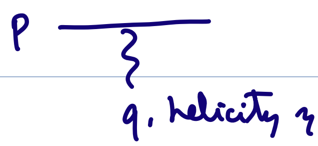
- Dirac quantisation : $e_1 g_2 - e_2 g_1 \in \frac{\hbar}{2}\mathbb{Z}$

  Gravity (?) : $\underbrace{u_1 \cdot u_2}_{=\gamma = \frac{1}{\sqrt{1-v^2}}} (m_1 n_2 - m_2 n_1) \in \frac{\hbar}{2}\mathbb{Z}$

  Perturbation theory : $\frac{\text{above}}{J} \ll 1$

- $A_4 =$  subtle (pairwise helicity, ...)

  But non-analytic part clear

- 3-pt amplitudes 

  - EM : $A_3 = (e + i\eta g)\, \epsilon_{(\eta)} \cdot p = \sqrt{e^2+g^2}\, e^{i\eta\theta}\, \epsilon_{(\eta)} \cdot p$

    $p_\mu = m u_\mu$

    U(1) duality : $\epsilon_{(\eta)} \mapsto e^{i\eta\psi} \epsilon_{(\eta)}$, $\theta \mapsto \theta - \psi$

    Fixes all $A(\ldots)$ $\to$ impulse , waveform to all orders

• Gravity: $A_3 = \frac{\kappa}{2} e^{i\eta\theta} (\epsilon_{(\eta)}\cdot p)^2 \qquad \varepsilon^{\mu\nu}_{(\eta)} = \epsilon^{\mu}_{(\eta)}\epsilon^{\nu}_{(\eta)}$

$p_\mu = M u_\mu \,, \quad M = \sqrt{m^2+n^2}\,, \quad m+in = M e^{i\theta}$

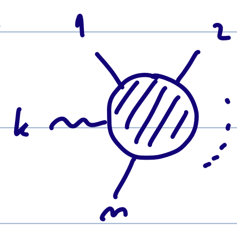
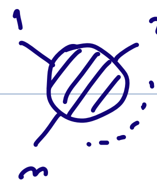
$U(1)$: $\epsilon_{(\eta)} \mapsto e^{i\eta\psi}\epsilon_{(\eta)} \,, \quad \theta \mapsto \theta - 2\psi$

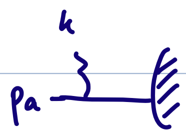
Broken in full theory.

Nutty amplitudes beyond A_3?

4 Soft theorems

Soft photon / graviton ($k\to 0$):

A_{n+1} () $\simeq S(k) \times A_n$ ()

Dominated by  : $\dfrac{1}{(p_a+k)^2 - m_a^2} = \dfrac{1}{2p_a\cdot k}$

• Usual EM : $S = \sum_{a=1}^{n} e_a \dfrac{\epsilon\cdot p_a}{k\cdot p_a} \qquad \left(\sum_a p_a^\mu = 0\right)$

gauge inv. $S(\epsilon_\mu \propto k_\mu) = 0 \Rightarrow \sum_{a=1}^{n} e_a = 0$

$\rightarrow$ charge conservation

- Dyonic EM: $$S_{(\eta)} = \sum_{a=1}^{m} (e_a + i\eta g_a) \frac{\epsilon_{(\eta)} \cdot p_a}{k \cdot p_a}$$

gauge inv. $\sum_{a=1}^{m} (e_a + i\eta g_a) = 0 \Rightarrow \sum_{a=1}^{m} e_a = \sum_{a=1}^{m} g_a = 0$

$\rightarrow$ both electric and magnetic charges conserved

- Usual gravity: $$S = \sum_{a=1}^{m} \frac{\kappa_a}{2} \frac{\varepsilon_{\mu\nu} p_a^\mu p_a^\nu}{k \cdot p_a}$$

gauge inv. $S(\varepsilon_{\mu\nu} = \xi_{(\mu} k_{\nu)}) = 0 \Rightarrow \sum_{a=1}^{m} \kappa_a p_a^\mu = 0$

$\sum_{a=1}^{m} p_a^\mu = 0 \quad \rightarrow \kappa_a = \kappa$ universal coupling

- Nutty gravity: $$S_{(\eta)} = \frac{\kappa}{2} \sum_{a=1}^{m} e^{i\eta\theta_a} \frac{\varepsilon_{\mu\nu}^{(\eta)} p_a^\mu p_a^\nu}{k \cdot p_a}$$

gauge inv. $\sum_{a=1}^{m} e^{i\eta\theta_a} p_a^\mu = 0$

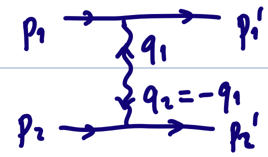
$\rightarrow \sum_{a=1}^{m} \cos\theta_a \, p_a^\mu = \sum_{a=1}^{m} \sin\theta_a \, p_a^\mu = 0$

If $p_a^\mu = M_a u_a^\mu$ $<$ $\cos\theta_a \, p_a^\mu = m_a u_a^\mu$, $\sin\theta_a \, p_a^\mu = n_a u_a^\mu$

But we only have $\sum_{a=1}^{m} p_a^\mu = 0$!

PUZZLE

Our goal: memory from $A_5^{(\eta)} \simeq S^{(\eta)} A_4$, $k = \hbar\omega(1, \hat{x})$

$A_4 =$ 

Try: $$S^{(\eta)} = \frac{\kappa}{2} \sum_{i=1}^{2} e^{i\eta\theta_i}\, \varepsilon^{(\eta)}_{\mu\nu} \Big(\frac{p_i'^{\mu} p_i'^{\nu}}{k\cdot p_i'} - \frac{p_i^{\mu} p_i^{\nu}}{k\cdot p_i} - \underbrace{\frac{q_i^{\mu} q_i^{\nu}}{k\cdot q_i}}_{\text{added}} \Big)$$

gauge inv. $\sum_{i=1}^{2} e^{i\eta\theta_i} \big(\underbrace{p_i'^{\mu} - p_i^{\mu} - q_i^{\mu}}_{=0}\big) = 0$ ✓

But $\frac{1}{k\cdot q_i}$ dangerous.

⑤ Memory

$$\Delta \varsigma_A = \frac{\mathcal{E}_{AB}(\hat{x})}{4\pi r} \varsigma^B$$

$$\mathcal{E}_{AB}(\hat{x}) = -4\pi r \int_{-\infty}^{+\infty} du \int_{-\infty}^{u} du'\, R_{\mu\nu\rho\sigma}(\overset{\text{retarded time}}{u'}, r, \hat{x})\, \underbrace{\tilde{l}^{\mu} e_A^{\nu} \tilde{l}^{\rho} e_B^{\sigma}}_{\text{celestial projection}}$$

$$= -\frac{\kappa^2}{4} \sum_{i=1}^{2} \left[\cos\theta_i\, E_{ABCD} - \sin\theta_i\, O_{ABCD}\right] D^C D^D \Phi_i(\hat{x})$$

cov. derivative S^2

- $E_{ABCD} = \gamma_{AC}\gamma_{BD} + \gamma_{AD}\gamma_{BC} - \gamma_{AB}\gamma_{CD}$ "electric" projection (γ_{AB}: S^2 metric)
- $O_{ABCD} = \epsilon_{AC}\gamma_{BD} + \epsilon_{BD}\gamma_{AC}$ "magnetic" projection

- memory potentials: $\quad l_\mu = (1, \hat{x})$

$$\Phi_i(\hat{x}) = \underbrace{l \cdot \Delta p_i \log(l \cdot p_i)}_{\text{usual}} - l \cdot \Delta p_i \underbrace{\log |l_\perp|}_{\text{diverges along direction of scattering}} + \text{finite piece}$$

- Gauge invariance vs regularity of $\mathcal{E}_{AB}(\hat{x})$ (unlike EM)
- "magnetic memory" signals NUT charge