Spin Universality to the Extreme

Spinning Black Holes from a Tower of fixed-Spin Theories

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BUAP - New Trends in QFT, Amplitudes and Gravity
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Introduction

Future Gravitational Wave Observatories

Future ground based Observatories
→ Advanced LIGO
→ Einstein Telescope
→ Cosmic Explorer

Future space based Observatories
→ LISA
→ TianQin
→ Taiji

Physics goals:
→ Tests of General Relativity
→ Black Hole Formation
→ Neutron Star Equation of State

Requires theoretical modeling of perturbative inspiral phase up to $\mathcal{O}(G^7)$

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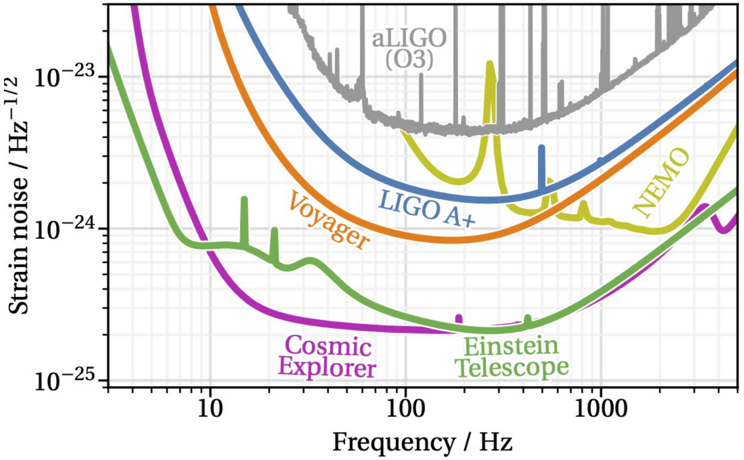
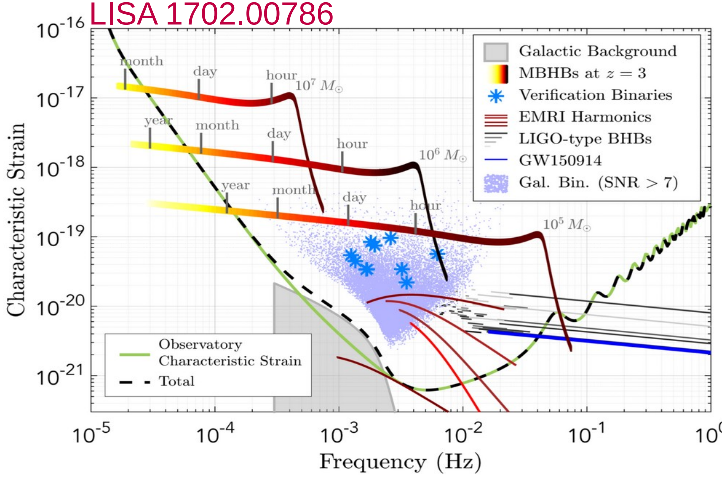


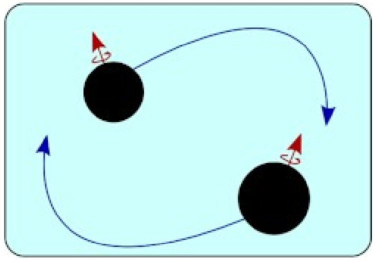
LISA 1702.00786



The Toolbox for Analytic Calculations

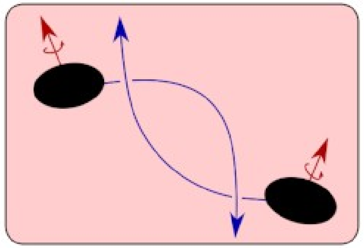
Post-Newtonian Expansion

- Large separation, slowly moving
- Bound orbits
- Expansion: $v^2/c^2 \sim GM/(rc^2) \ll 1$



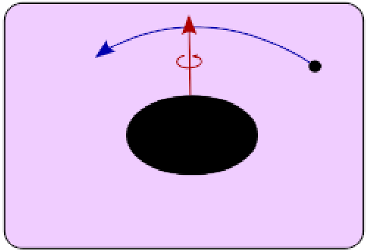
Post-Minkowskian Expansion

- Large separation, fast moving
- Open orbits
- Expansion: $GM/(rc^2) \ll 1$



Gravitational Self-Force

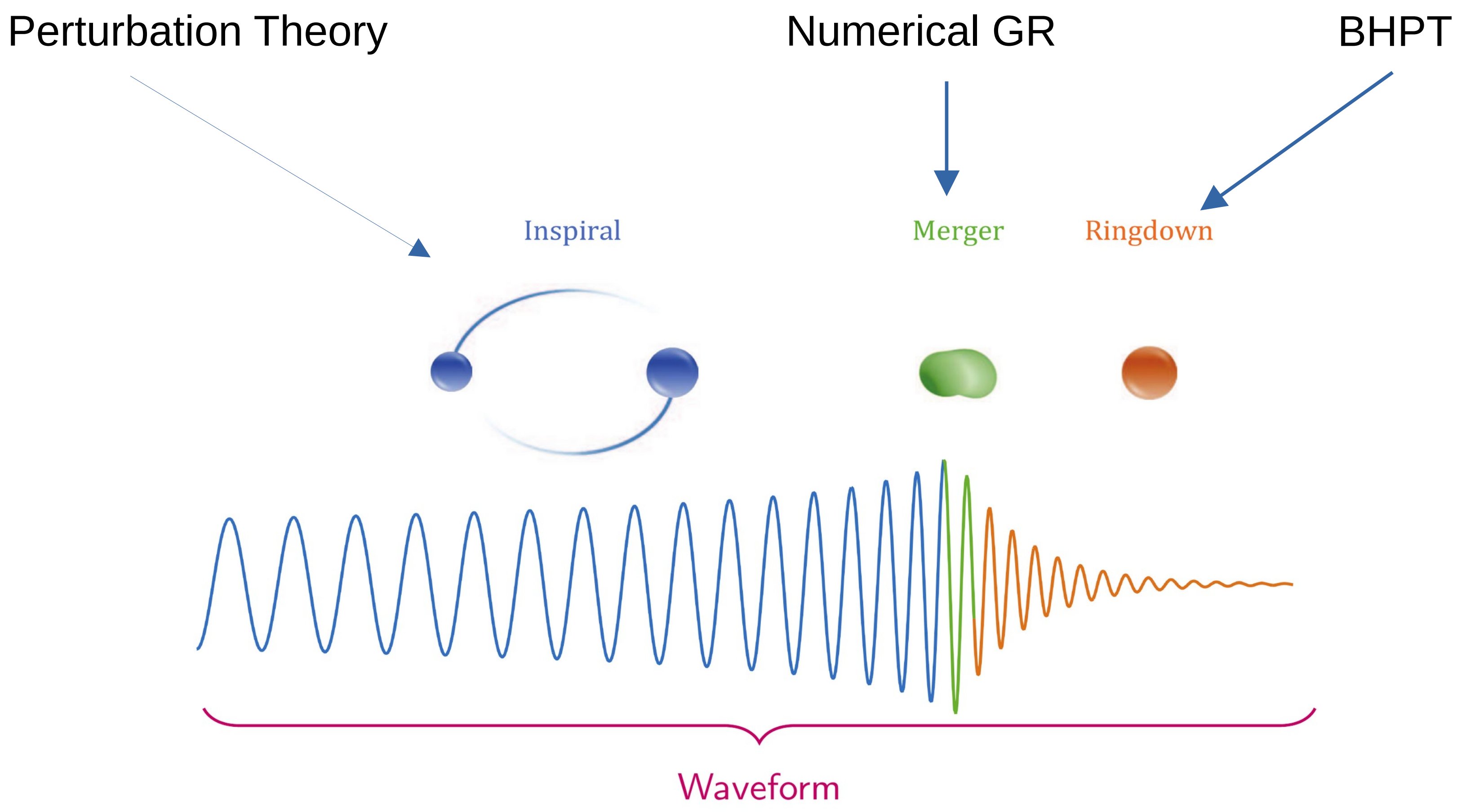
- All-orders in G
- Expansion: $m_2/m_1 \ll 1$



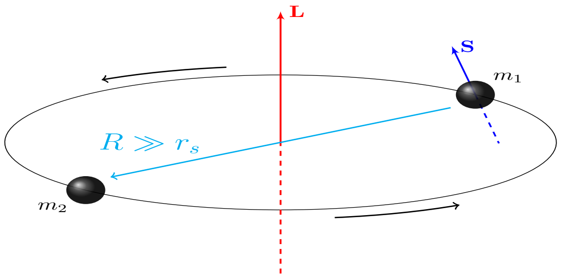
Amplitudes are natural for post-Minkowskian expansion!

Stages of a Binary Merger

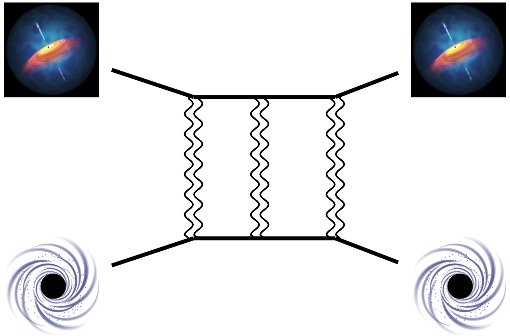
Perturbation Theory | Numerical GR | BHPT



Point Particle Effective Field Theory



[Goldberger, Rothstein '06]



Massive particles coupled to Gravity!

- Lagrangian:

$$\mathcal{L} = \mathcal{L}_{\text{EH}} + \mathcal{L}_{\text{point-particle}} + \mathcal{L}_{\text{finite-size}}$$

- Finite-size effects via tidal operators:

$$\mathcal{L}_{\text{finite-size}} \subset \sum c_{abc} \phi_i \nabla^a \phi_i \nabla^b R^c$$

- Spin DoF by including massive spinning fields

$$\phi_i \to \phi_i^{\mu_1 \cdots \mu_N}$$

General Relativity as a QFT

- The Action

$$S[\phi, g_{\mu\nu}] = \int d^4x\sqrt{-g}\left[-\frac{2}{\kappa^2}R + \frac{1}{2}\sum_{i=1}^{2}\left(g^{\mu\nu}(\partial_\mu\phi_i)(\partial_\nu\phi_i) - m_i^2\phi_i^2\right)\right]$$

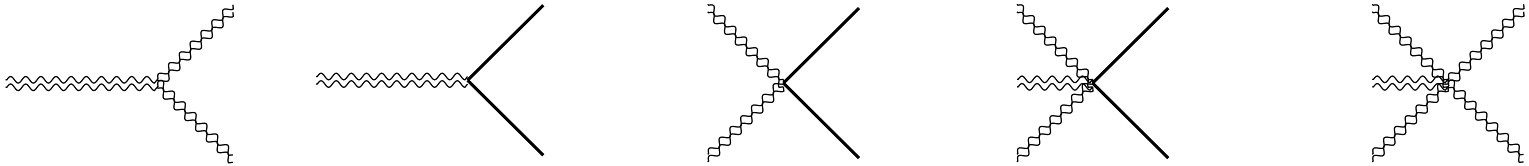
- Expand spacetime around flat space

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa\, h_{\mu\nu} \qquad\qquad \kappa = \sqrt{32\pi G}$$

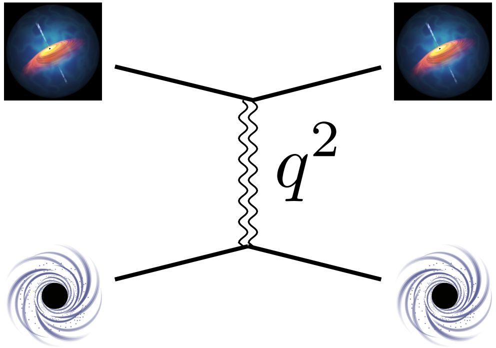
- General Relativity is non-linear

$$g^{\mu\nu} = \eta^{\mu\nu} - \kappa\, h^{\mu\nu} + \kappa^2\, h^{\mu\lambda}h^{\nu}{}_{\lambda} + \ldots$$

- Generates infinite many Feynman Rules



QFT at work



$$A^{\text{tree}} = \frac{4\pi G}{E_1 E_2} \frac{m_1^2 m_2^2 (1-2\sigma^2)}{q^2} + \cdots$$

The classical potential is given by

$$V(r) = \int \frac{d^3 q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{r}} A^{\text{tree}} = -\frac{G m_1 m_2}{r}\left(\frac{m_1 m_2}{E_1 E_2}(2\sigma^2 - 1)\right)$$

$$= -\frac{G m_1 m_2}{r} + \mathcal{O}(v^2)$$

- $q \to 0$ reveals the classical physics
- Amplitudes provide full special relativistic corrections
- Loop-Amplitudes provide higher-order $\mathcal{O}(G^n)$ corrections

Structure of classical amplitudes

$$\mathcal{M}^{\text{tree}} = G\left(\frac{c_0^{cl}}{q^2} + \dots\right.$$

$$\mathcal{M}^{\text{1-loop}} = G^2\left(\frac{c_1^{scl}}{q^2} + \frac{c_1^{cl}}{|q|} + c_1^{Q}\log(q^2)\dots\right)$$

$$\mathcal{M}^{\text{2-loop}} = G^3\left(\frac{c_2^{sscl}}{q^2} + \frac{c_2^{scl}}{|q|} + c_2^{cl}\log(q^2)\dots\right)$$

- Classical terms at **any** loop order
- Super-classical terms have to cancel in physical observables!
- Quantum corrections are suppressed by powers of $\hbar$

The Amplitude-Action Relation

- For conservative physics, everything is encoded in the Radial Action

[Bern, Parra-Martinez, Roiban, Ruf, Shen, Solon, Zeng '21]

$$i\mathcal{M} = |\mathbf{p}| \int d^{D-2}\mathbf{b}\, e^{i\mathbf{b}\cdot\mathbf{q}} \left(e^{iI_r} - 1\right)$$

3-momentum at infinite past

Impact parameter

Momentum transfer

- Expanding $\mathcal{M}$ and I_r in G we obtain

$$\mathcal{M}^{(1)} = \tilde{I}_r^{(1)}$$

$$\mathcal{M}^{(2)} = \tilde{I}_r^{(2)} + \int_\ell \frac{\tilde{I}_r^{(1)}\tilde{I}_r^{(1)}}{Z_1}$$

$$\mathcal{M}^{(3)} = \tilde{I}_r^{(3)} + \int_\ell \frac{\tilde{I}_r^{(2)}\tilde{I}_r^{(1)}}{Z_1} + \int_\ell \frac{\tilde{I}_r^{(1)}\tilde{I}_r^{(1)}\tilde{I}_r^{(1)}}{Z_1 Z_2}$$

Iteration integrals

Superclassical divergences cancel with Iteration integrals

Spinning Black Holes

Amplitude Approaches [Slide stolen from Mao Zeng]

- Massive spinor helicity amplitudes (spin exponentiation) [Arkani-Hamed, Huang, Huang, ‘17 …] High ranks for tensor integrals, difficult IBP reduction beyond one-loop

- Non-transverse higher-spin fields [Bern, Luna, Roiban, Shen, Zeng ’20, Alaverdian, Bern, Kosmopoulos, Luna, Roiban, Scheopner, Teng ‘24] Needs separation between spin-magnitude changing effects and conservative dynamics; not yet achieved beyone one-loop

- **Finite-spin massive particles** [Holstein, Ross ‘07, Vaidya ‘14, Maybee, O’Connell, Vines ‘19, Damgaard, Haddad, Helset ‘19, Aoude, Haddad, Helset ‘20]
  → Straightforward option for > 1-loop calculations
  → Spin-j accesses $\mathcal{O}(S^{2j})$ perturbative multipole moments
  → Plagued by Casimir ambiguity, resolved by spin-interpolation

- Beyond amplitudes: PMEFT with spin [Liu, Porto, Yang], WQFT [Jakobsen, Mogull, Plefka, Steinhoff ‘21, Haddad, Jakobsen, Mogull, Plefka ‘24], Generalized Wilson Lines [Bonocore, Kulesza, Pirsch ‘25]

State-of-the-art for Spin effects

- High orders for spin at 1-loop [Bern, Kosmopoulos, Luna, Roiban, Teng ‘22; Aoude, Haddad, Helset ‘22,’23; Bohnenblust, Cangemi, Johannson, Pichini ‘24]
- Gravitational Waveform at $\mathcal{O}(G^3 S^2)$ [Bohnenblust, Ita, MK, Schlenk ‘23,’25]
- Conservative + dissipative effects in WQFT at $\mathcal{O}(G^3 S^2)$ [Jakobsen, Mogull, ‘22]
- All-order-in-spin radiation reaction [Alessio, Di Vecchia, ‘22]
- Energy loss from PMEFT [Riva, Vernizzi, Wong ‘22]
- Three-loop in WQFT $\mathcal{O}(G^4 S^1)$ [Jakobsen, Mogull, Plefka, Sauer, Xu ‘23]
- Conservative dynamics at $\mathcal{O}(G^3 S^2)$ from amplitudes [Febres Cordero, MK, Lin, Ruf, Zeng ‘22; Akpinar, Febres Cordero, MK, Ruf, Zeng ‘24]
- Unprecedented results at $\mathcal{O}(G^3 S^4)$ [Akpinar, Febres Cordero, MK, Smirnov, Zeng ‘25]

Spin Universality and Ambiguity

- Fixed spin-j amplitudes have access to $\mathcal{O}(S^{2j})$ multipoles

$$\mathcal{A}(\phi_1\phi_2 \to \phi_1\phi_2) \sim \mathbb{1}, \mathcal{A}(\phi\psi \to \phi\psi) \sim \{\mathbb{1}, S\cdot l\},$$
$$\mathcal{A}(\phi V \to \phi V) \sim \{\mathbb{1}, S\cdot l, (q\cdot S)^2, (p\cdot S)^2\}$$

- Spin Universality: (conjecture!)
  Classical multipole moments do not depend on the nature of the test particle

- For arbitrary spin there should be a fifth spin-operator

$$O_5 = q^2 S^2 \sim \frac{\hbar^2}{\hbar^2} = 1 \,, \qquad O_5^{\text{spin-j}} = q^2 S^2 = q^2 j(j+1)\mathbb{1}$$

- 1-Loop example: Classical physics lives at 1/|q| and quantum physics at |q|

$$\mathcal{A} = \frac{1}{|q|}\left[c_0\mathbb{1} + c_5(q^2S^2) + \ldots\right] = \frac{1}{|q|}\left[c_0 + c_5 q^2 j(j+1)\right]\mathbb{1}$$

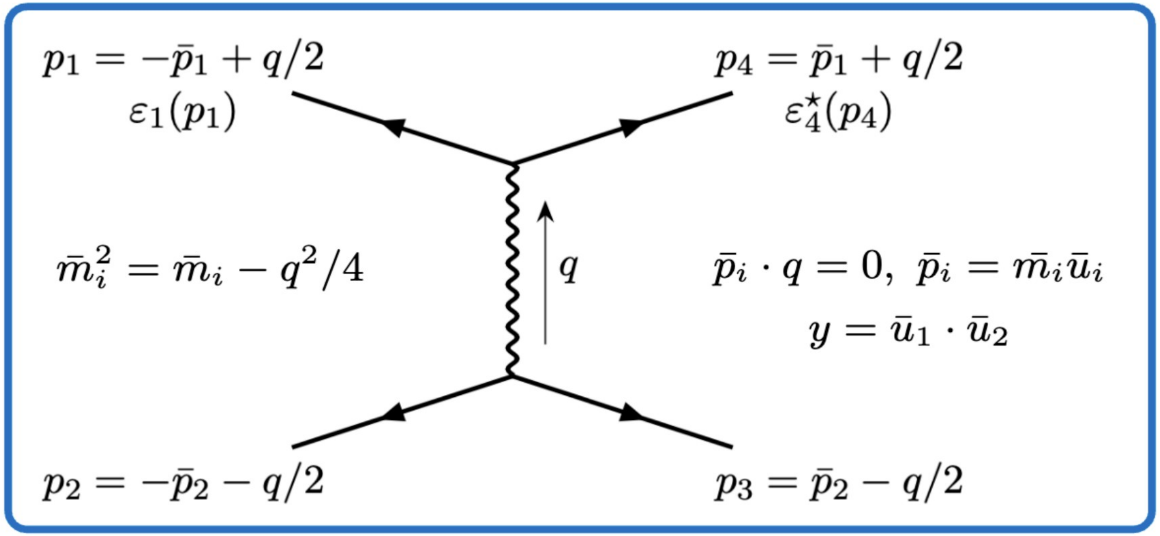
- Casimir operator indistinguishable from Quantum physics!

$\rightarrow$ Spin Interpolation!

A Tower of Spin Theories

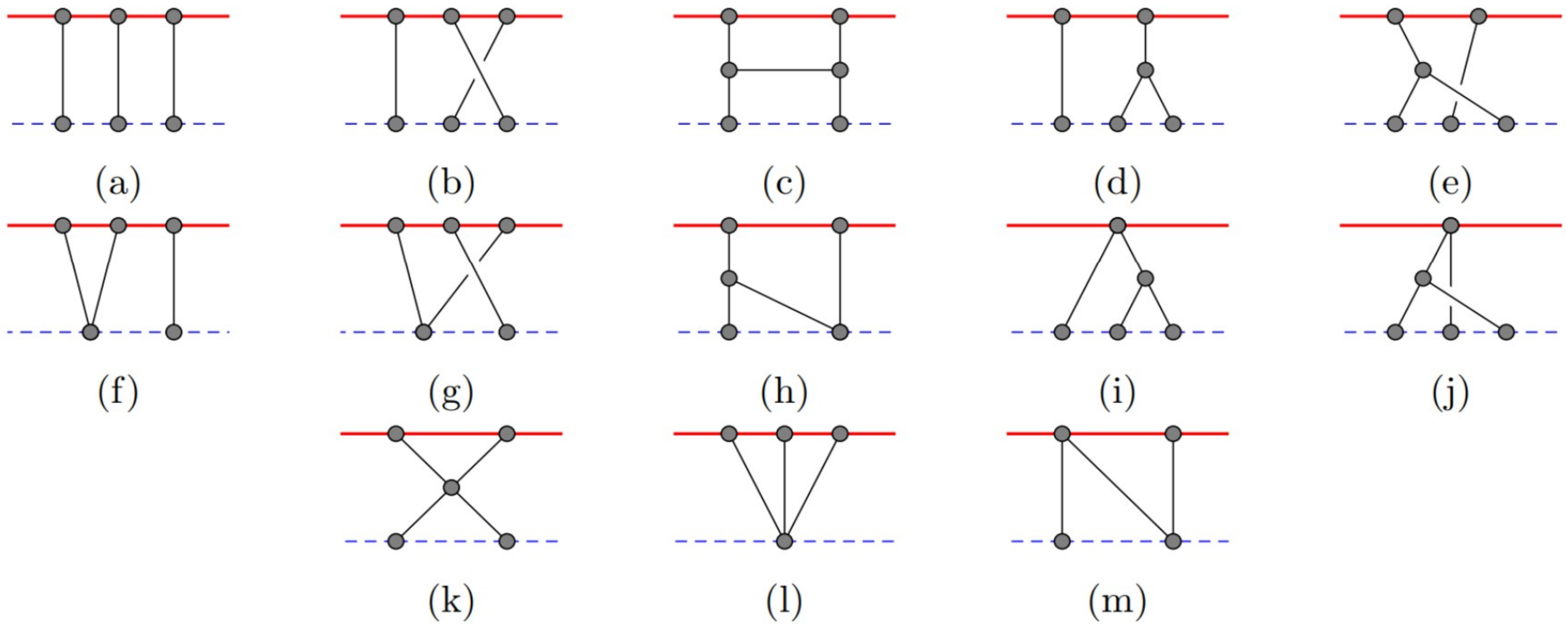
- Consider the following Scattering Processes

$$X(p_1,\epsilon_1) + \phi(p_2) \to \phi(p_3) + X(p_4,\epsilon_4) \qquad X = \{\Phi, V^\mu H^{\mu\nu}\}$$



$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{2R}{\kappa^2} + \frac{1}{2}g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - \frac{1}{2}m_2^2\phi^2 + \mathcal{L}_{m_1}$$

A Tower of Spin Theories



- Soft expansion [Parra-Martinez, Ruf, Zeng ‘20] followed by IBP reduction
  → Integrals with up to rank 13 and 10 dots, using private FIRE [Smirnov, Zeng]

A Tower of Spin Theories

[Abreu, Dormans, Febres Cordero, Ita, Kraus, Page, Pascual, S. Ruf, Sotnikov, '10]…..

Caravel

Feynman rules from $\mathcal{L}$

Numerical 3-point, 4-point and 5-point tree level amplitudes

Numerical agreement with analytical results obtained with double copy

Generalised Unitarity

Potential region $\longrightarrow$ no radiation!

Numerical tree-level amplitude + 1-loop and 2-loop integrands

Agreement with analytic tree-level and 1-loop amplitudes

Soft expansion + private FIRE6 code

Numerical tree-level, 1-loop and 2-loop amplitudes

4 form factors for spin-1
9 form factors for spin-2

Form factor decomposition of spinning amplitude

$$\mathcal{M} = \sum_n M_n \, \varepsilon^{\star}_{4\mu\nu\ldots} T_n^{\mu\nu\alpha\beta\ldots} \varepsilon_{1\alpha\beta\ldots}$$

Theory dependent information

Spin information

Spin in Quantum Field Theory

- Spin operator for massive spin-s boson

$$\mathbb{S}^{\mu}_{ij} = \frac{(-1)^s}{2m}\epsilon^{\mu\lambda\alpha\beta}p_\lambda \epsilon^{\star}_{iI(s)}(p)\left(\Sigma_{\alpha\beta}\right)^{I(s)}{}_{J(s)}\,\epsilon_j^{J(s)}(p)$$

- With the Lorentz-Generators in the spin-s representation

$$\left(\Sigma_{\alpha\beta}\right)^{I(s)}{}_{J(s)} = 2is\delta^{(I1}_{[\alpha}\eta_{\beta](J1}\delta^{I_2}_{J_2}\cdots\delta^{I_s)}_{J_s)}$$

- The classical spin vector can be seen as the expectation value

$$S^\mu = \sum_{i,j}\omega_i^\star \mathbb{S}^\mu_{ij}\omega_j \qquad \sum_i |\omega_i|^2 = 1$$

- Products of classical spin vectors → Symmetric products of spin operators

$$S^\mu S^\rho \cdots S^\sigma S^\nu = \sum_{i,k,\ldots}\omega_i^\star \mathbb{S}^{(\mu}_{ik}\mathbb{S}^{\rho}_{kj}\ldots\mathbb{S}^{\sigma}_{rm}\mathbb{S}^{\nu)}_{mj}\omega_j$$

Spin Interpolation - I

- Write Ansatz for the amplitude valid for generic spin representation

$$\{1, i(n\cdot S), (q\cdot S)^2, q^2 S^2, q^2(\bar{p}_2\cdot S)^2\} \quad n^\mu = \epsilon^{\mu\nu\alpha\beta} q_\nu \bar{p}_{1\alpha}\bar{p}_{2\beta}$$

- Ansatz for the finite part of the amplitude reads

$$\mathcal{M}^{(L)}_{fin} = \frac{\kappa^{2+2L}}{(-q^2)^{1-L/2+L\epsilon}} \sum_{n=0}^{4}\sum_{m} \alpha^{(L,n,m)}\mathcal{O}^{(n,m)}$$

- Spin Universality
  $\rightarrow$ spin coefficients (classical AND quantum) are independent of spin representation!

- Spin structures $\mathcal{O}^{(n,m)}$ organized in terms of

$$\Omega_\pm = (q\cdot S)^2 \pm q^2 S^2$$

Spin Interpolation - II

- To fix the coefficients we evaluate the Ansatz for $s = 0, 1, 2$ and expand up to $\mathcal{O}(q^4)$

  → 14 classical coefficients, given by the leading q contribution of each structure
  → 6 quantum suppressed coefficients

- For each representation, project onto the appropriate form factor basis

$$\mathcal{A} = \sum_n M_n \epsilon^{\star}_{4,\mu\nu,\ldots} T_n^{\mu\nu\alpha\beta\ldots} \epsilon_{1\alpha\beta\ldots}$$

- Solve a linear system of equations by demanding equality with the corresponding spinning amplitude

  → 31 linear relations, of which 11 are linearly dependent
  → 20 linearly independent relations fixes the 1+5+14=20 spin coefficients

Obtained for the first time results at $\mathcal{O}(G^3 S^4)$

Spin-Shift Symmetry

- Spin-Shift Symmetry states the Amplitude is invariant under

$$S^\mu \to S^\mu + \xi q^\mu$$

One-loop conjecture
[Bern, Kosmopoulos, Luna, Roiban, Teng '22]
[Aoude, Haddad, Helset '22]

- The only possible way to organize Spin multipoles

$$c_+\Omega_+ = c_+\left[(q\cdot S)^2 + q^2S^2\right] \Rightarrow c_+ = 0$$
$$c_-\Omega_- = c_-\left[(q\cdot S)^2 - q^2S^2\right] \Rightarrow c_- \neq 0$$

- The two-loop quartic in spin amplitude organizes itself as

$$\alpha^{(2,n,m)} = \alpha^{(2,n,m)}_{\text{probe}} + m_1^3 m_2^3 \alpha^{(2,n,m)}_{\text{1SF}}$$

Spin-Shift symmetric

Spin-Shift Symmetry breaking

First time spin-shift symmetry observed up to $\mathcal{O}(G^3S^4)$

The radial action and Observables

- Extract the radial action from the finite part of the Amplitude:

$$I_r = \int \frac{d^4q}{(2\pi)^2}\delta(p_1\cdot q)\delta(-p_2\cdot q)e^{ib\cdot q}\mathcal{M}_{fin}$$

Agreement with [Gonzo, Shi, '24] in the limit of a spinless probe in a Kerr background to quartic order in spin

- Dirac Brackets to compute observables

Following [Gonzo,Shi '24]

$$\Delta\lambda^\mu = \sum_j \frac{1}{j!}\underbrace{\{I_r,\{I_r,\cdots,\ldots,\{I_r,\lambda^\mu\}\ldots\}}_{j\ \text{times}} \quad \text{for} \quad \lambda^\mu = \{u_1^\mu, u_2^\mu, a_1^\mu, a_2^\mu\}$$

$$a_i = S_i/m_i$$

$$\{b^\mu, u_i^\nu\} = -\,\mathrm{sgn}_i\frac{(y^2-1)b^\mu b^\nu + l^\mu l^\nu}{m_i b^2(y^2-1)}\,,\quad \{b^\mu, a_i^\nu\} = \mathrm{sgn}_i\frac{u_i^\nu((y^2-1)b^\mu(b\cdot a_i)+l^\mu(l\cdot a_i))}{m_i b^2(y^2-1)}$$

$$\{b^\mu, b^\nu\} = \frac{(b^2(yu_2^\mu - u_1^\mu) - l^\mu(a_1\cdot u_2))b^\nu - (\mu\leftrightarrow\nu)}{m_1 b^2(y^2-1)} + (1\leftrightarrow 2, b\to -b)\,,\quad \{a_i^\mu, a_i^\nu\} = \frac{\epsilon^{\mu\nu}{}_{\rho\sigma}u_i^\rho a_i^\sigma}{m_i}$$

Validation of our results

- Agreement with the aligned-spin scattering angle in **post-Newtonian results** [Bautista, Khalil, Sergola, Kavanagh, Vines, ‘24] through $\mathcal{O}(G^3S^4)$

- Conservative observables from Dirac Brackets agree with our previous $\mathcal{O}(G^3S^2)$ results from **KMOC** [Febres Cordero, MK, Lin, Ruf, Zeng ‘22] and with **WQFT** results [Jakobsen, Mogull ‘22]

- Using all-order-in-spin radiation reaction amplitudes [Alessio, Di Vecchia ‘22] and acting with Dirac Brackets, obtain **cancellation of high-energy divergences** in observables through $\mathcal{O}(G^3S^4)$, previously observed through $\mathcal{O}(G^3S^2)$ using WQFT [Jakobsen, Mogull ‘22]

- Strong and highly non-trivial checks provide confidence in our partly conjectural setup!

Conclusions & Outlook

- First calculation of $\mathcal{O}(G^3 S^4)$ conservative dynamics of binary Kerr Black Hole using scattering amplitudes from two-loop numerical unitarity!

  What is next?
  $\rightarrow$ Disspative effects?
  $\rightarrow$ Higher-loop orders?
  $\rightarrow$ Higher spin?

- What if we are interested in Neutron stars?

  $\rightarrow$ Higher-dimensional Operators
  $\rightarrow$ How to do Spin interpolation?

- What is the origin of the Spin-Shift symmetry?!