Causality constraints on effective theories of gravity

Simon Caron-Huot

McGill University

On work with:

Vincent van Duong
Yue-Zhou Li
Julio Parra Martinez
Dalimil Mazac
Leonardo Rastelli
David Simmons-Duffin

+ in progress with Celina Pasiecznik, Julia Pasiecznik & Junsei Tokuda

New Trends in Quantum Field Theory 2026
(CIIEC Puebla, México; virtually)

Effective field theories

A powerful tool to focus on the physics at a given scale,
parametrizing possible effects from shorter-distance physics.

-Fluids on lengths $\ll \lambda_{\rm mfp}$: focus on pressure, bulk velocity, density.
forget $\sim 10^{23}$ positions

-QED at energies $\ll m_e$: keep electron but not positron

-Electroweak @ $\ll m_W$: light fermions + four-fermion interaction

...

In these examples, short-distance physics is known: EFT is a tool.

Today: situations where we do *not* know the microscopics!

Effective field theories (II)

Systematic parametrization of effects of short-distance physics on long distances:

-Beyond-Standard-Model searches: $\mathscr{L}_{\rm SM} + \mathscr{L}_{\nu-\rm masses} + ???$

-Gravity at long distances: $\mathscr{L}_{\rm GR} + \mathscr{L}_{SM}^{\rm minimal\,coupling} + ???$

Higher-dimension ("irrelevant") operators:
$F^3, (H^\dagger DH)^2, \mathrm{Riem}^2, \ \ldots)$

Q: What options are theoretically consistent?

- Could new physics be hidden from first few orders and only appear at eg. $(D^{10}H)^2$?
- Could gravity be modified at long distances (eg. km-scale black hole mergers) without affecting particle physics?

Outline

1. Relativistic Causality
   - The architect of quantum field theory!
   - Causality for scattering amplitudes

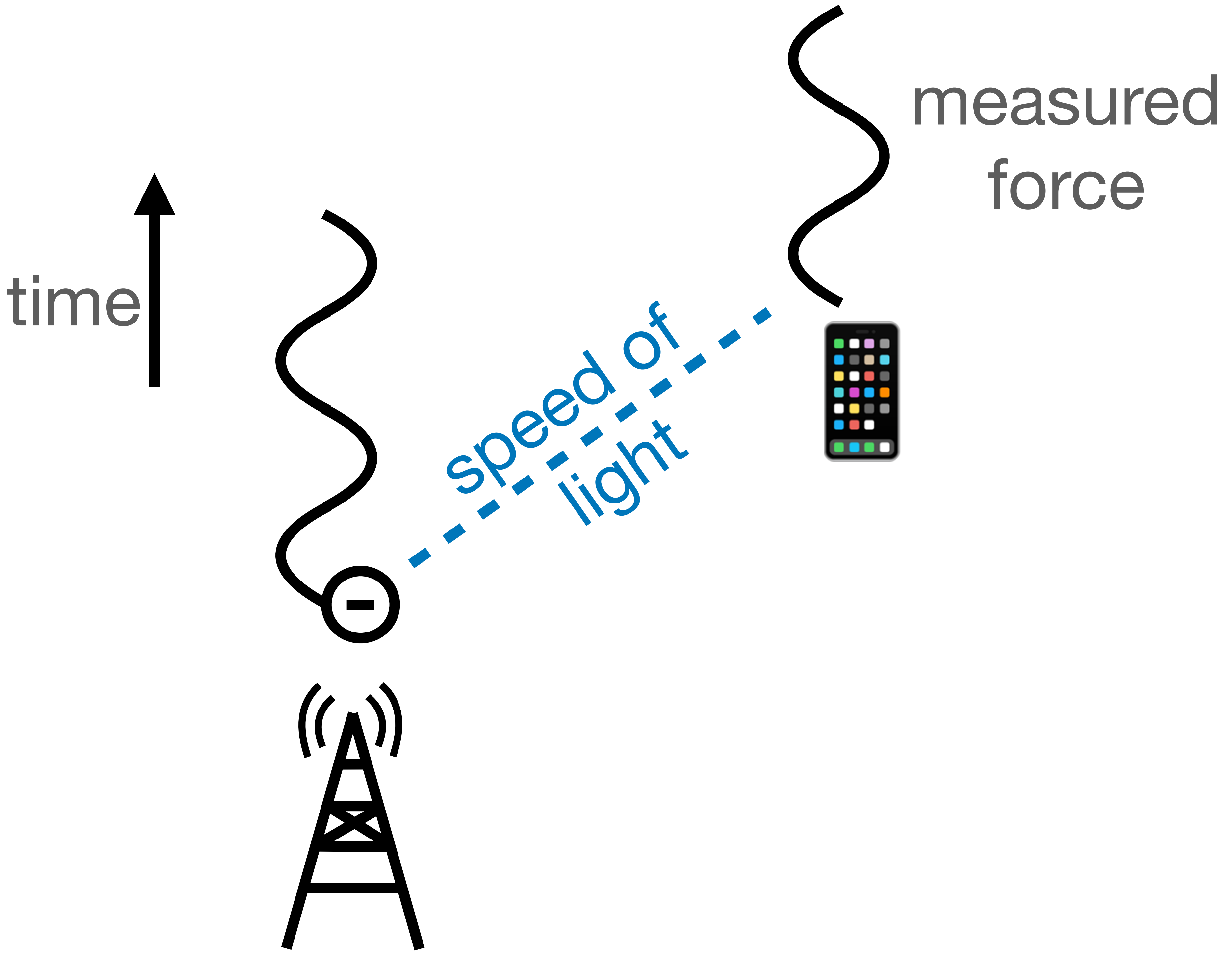
2. Constraints on effective theory
   - Single scalar: method and results
   - What is special about gravity?

3. Discussion
   - What is allowed? what could be constrained next?

Relativistic causality

"signals can't travel faster than light"

- Why waves, fields



Relativistic causality

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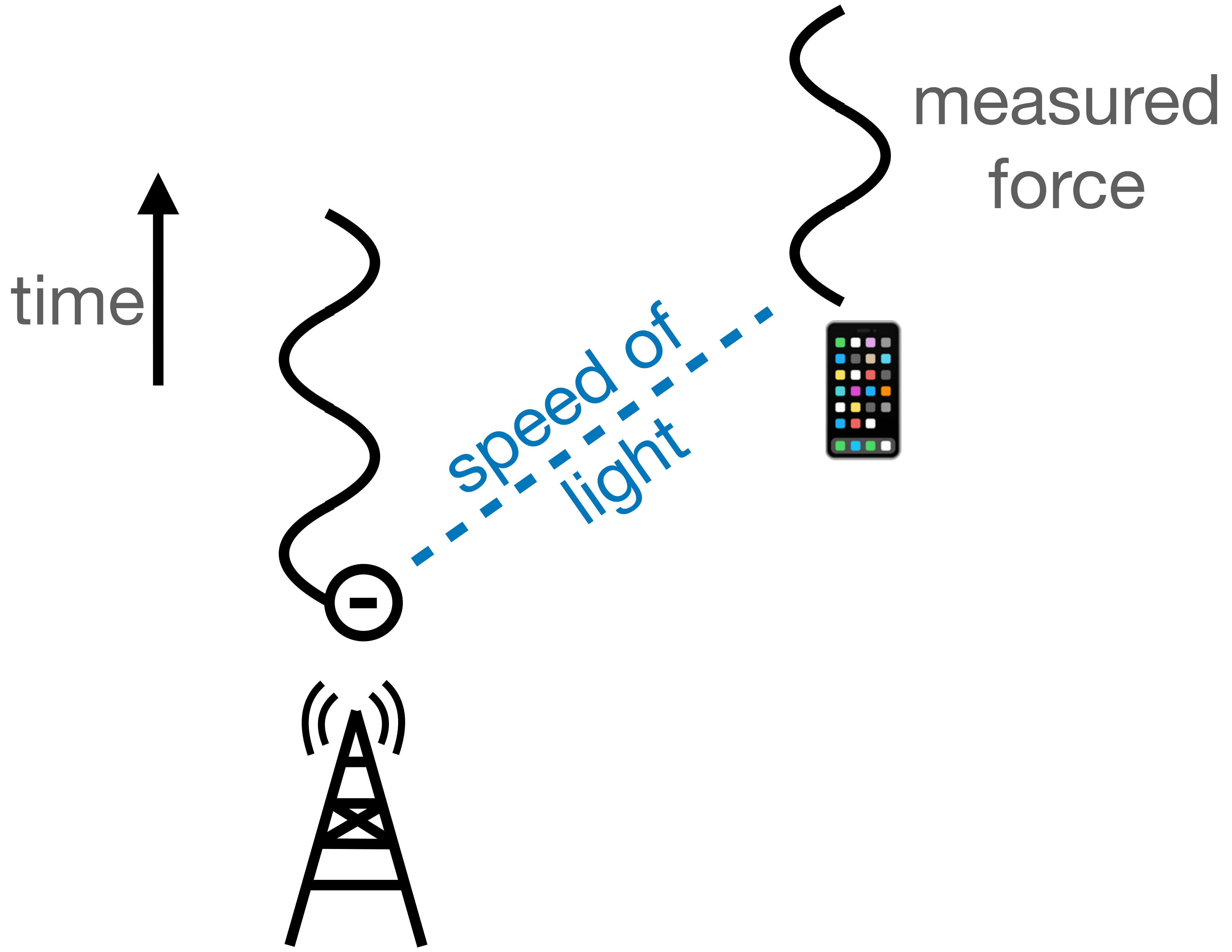
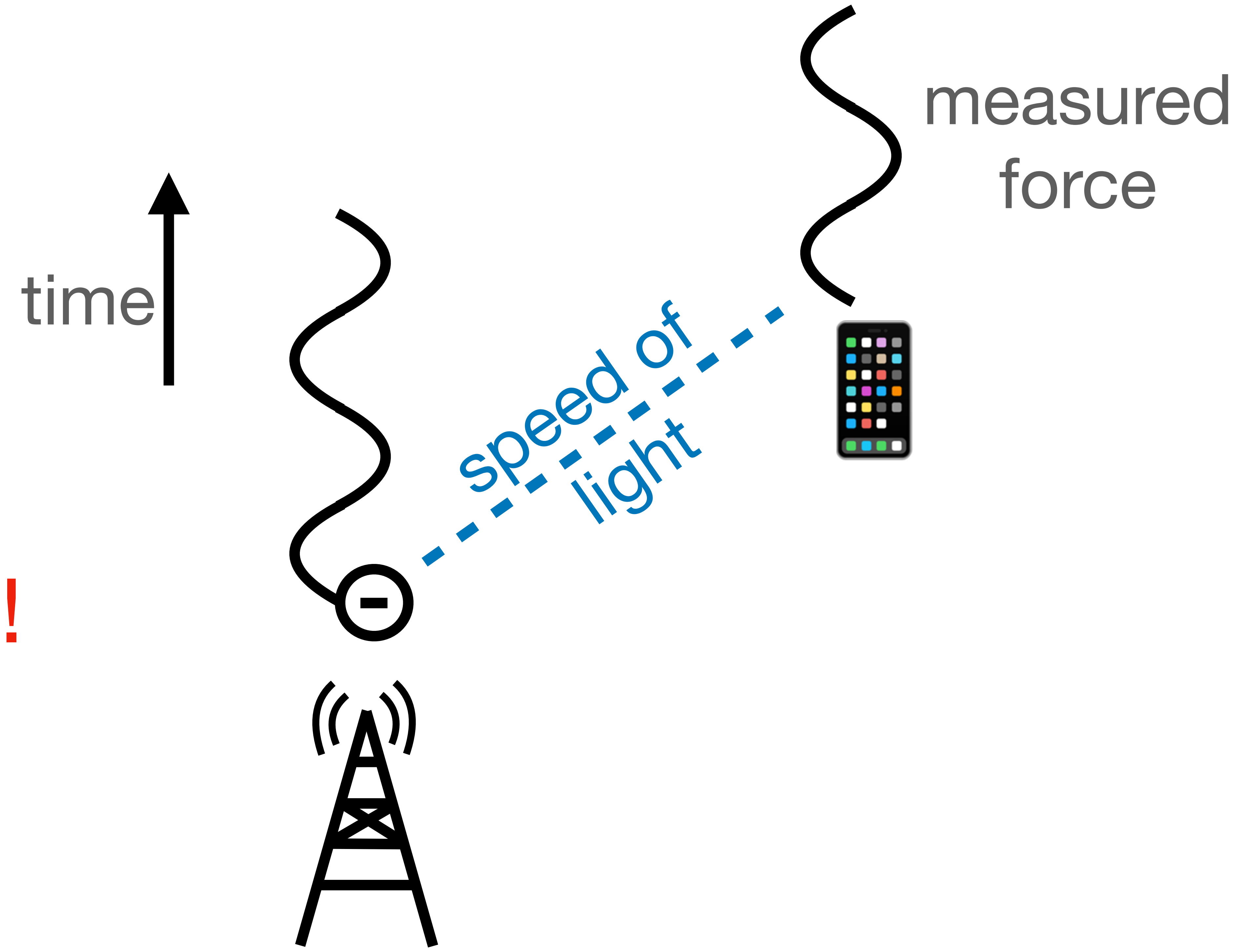


Relativistic causality

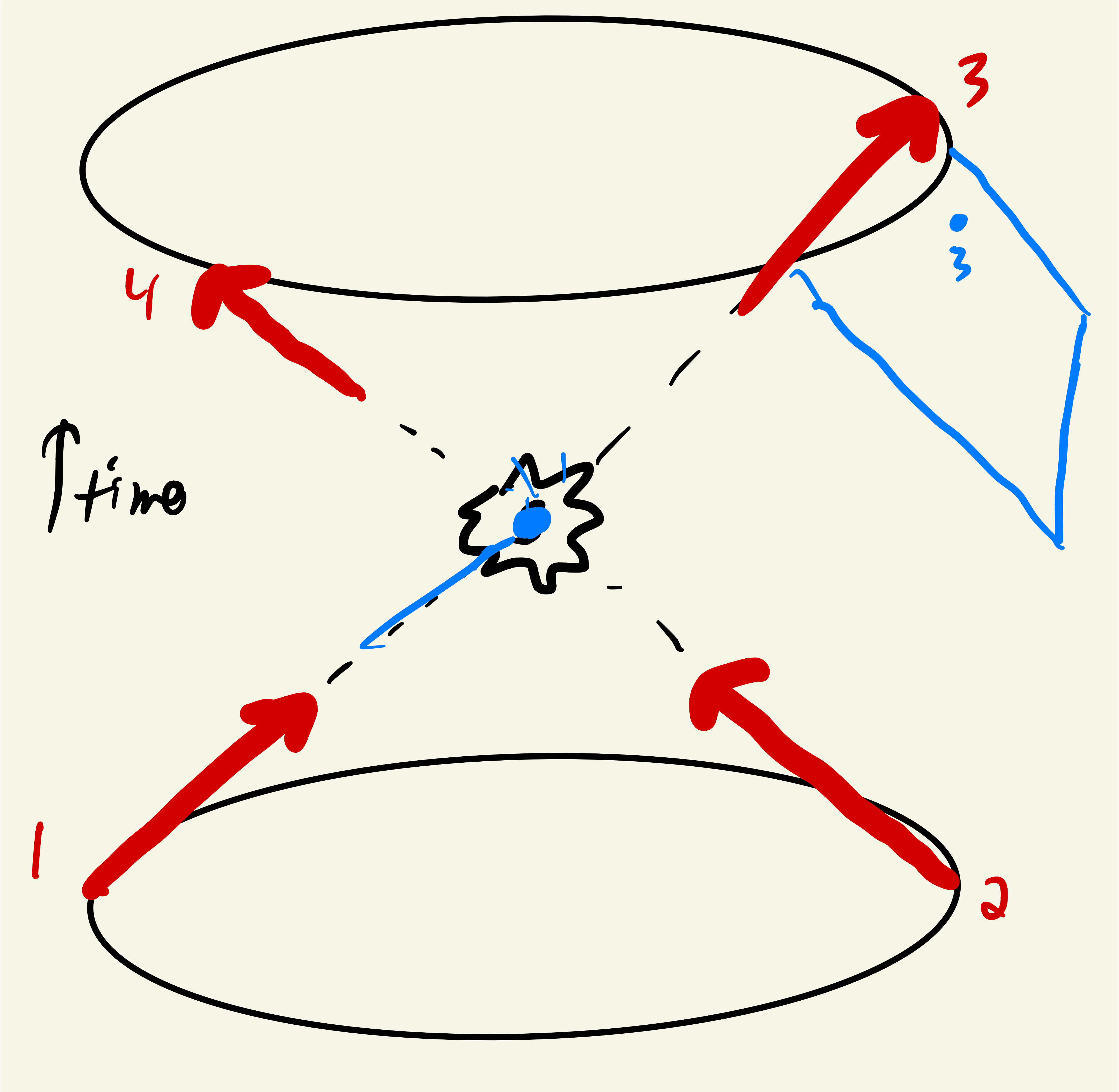
"signals can't travel faster than light"

- Why waves, fields
- Why particles
- Why antiparticles
- ...
- Why EFTs have to work
- Why gravity is attractive
- ...

the architecture of QFT!!



Causality for 2->2 scattering



- Fixed-angle scattering is not sharply constrained

[think billiard balls; cf Mizera 2306.05395]

- Key is ultrarelativistic, near-forward kinematics:

$$s \to \infty$$
$$t \text{ or } b \text{ fixed}$$

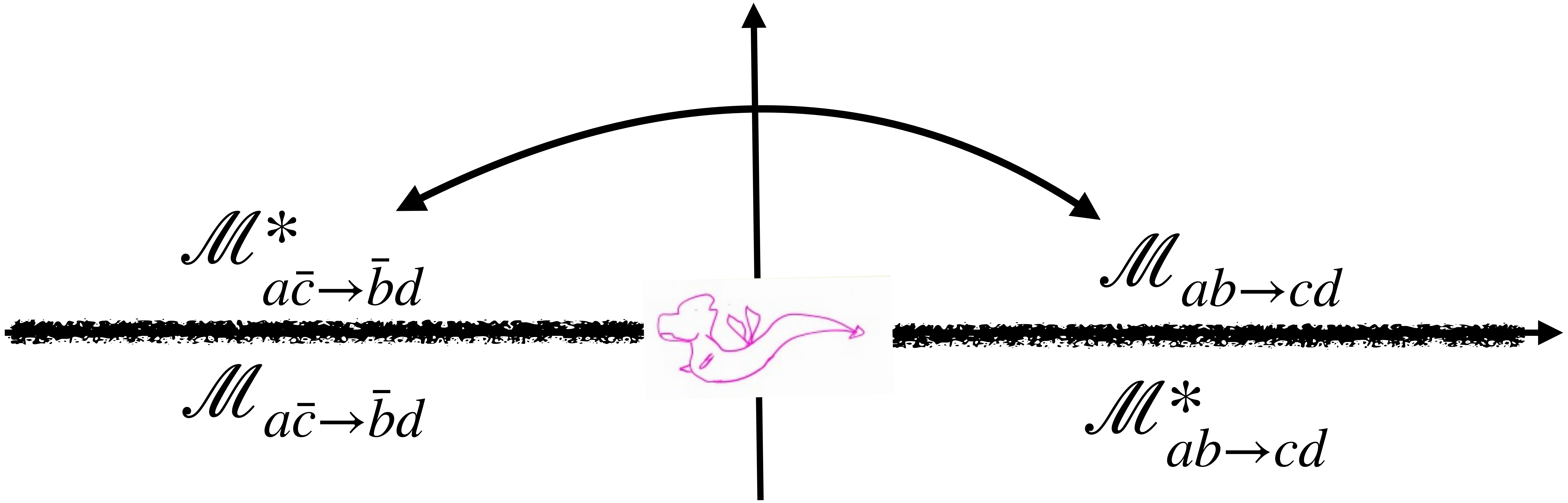
("Regge limit")

- Related to crossing:

particle $1 \to 3 \simeq$ antiparticle $3 \to 1$

Causality $\simeq$ analyticity in upper-half-plane

$\lfloor s$
fixed t



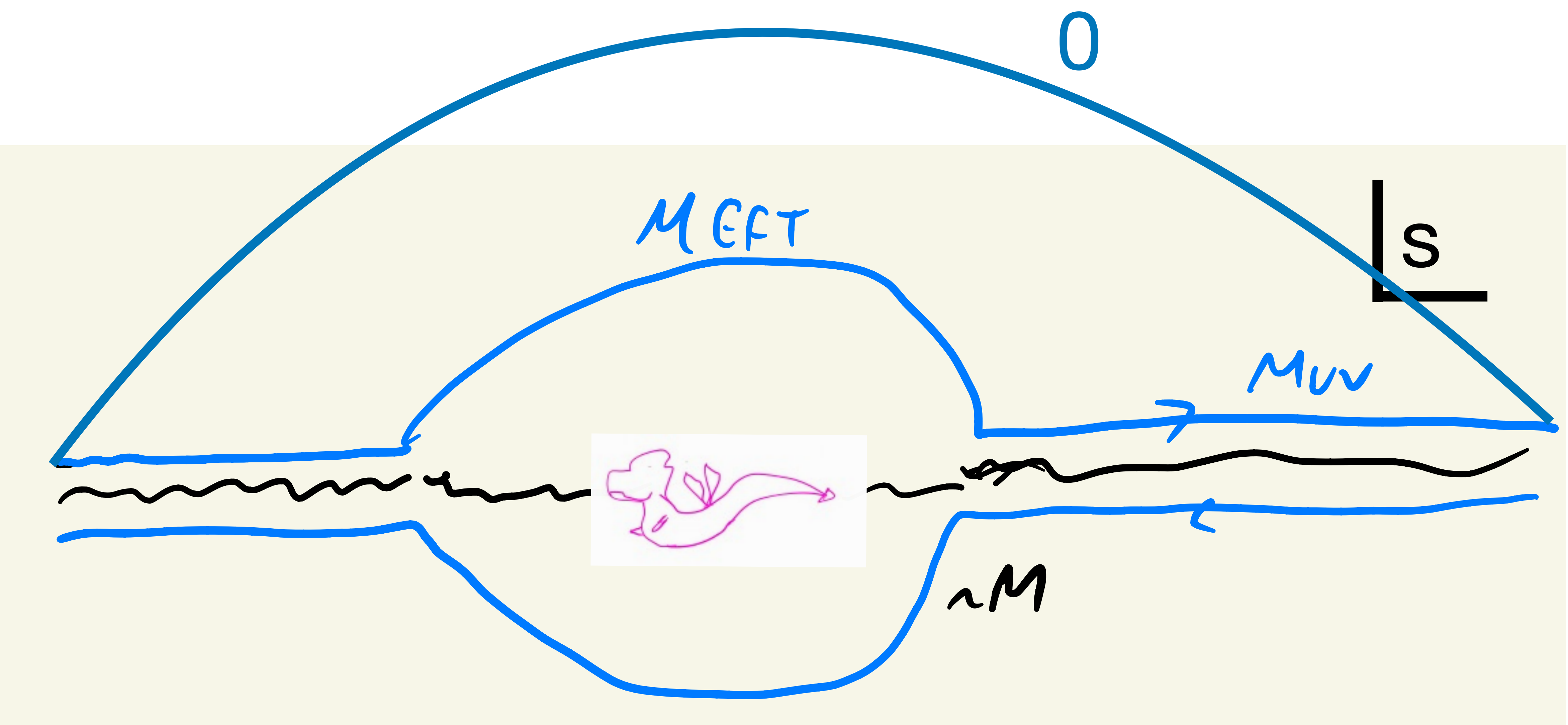
Unitarity bounds amplitudes: $\langle \mathrm{out}\,|\,\mathrm{in}\rangle \leq |\langle \mathrm{out}\,|\,\mathrm{out}\rangle\langle \mathrm{in}\,|\,\mathrm{in}\rangle|^{1/2}$

$\Rightarrow$ "Froissart Lemma": $\displaystyle\int_t \psi(t)\mathscr{M}(s,t) \leq |s| \times \mathrm{const}$ through upper-half-plane

(mass gap not needed)

[cf discussion in 2201.06602;
my 2025 TASI lectures (recorded)]

Dispersive sum rules
à la Kramers-Kronig



Relation between IR and UV:

$$0 = \oint_{|s|=\infty} ds \frac{\mathcal{M}_\psi(s)}{s^{k+1}} \quad \Rightarrow \quad \oint_{\text{EFT}} (\cdots) = \int_{\text{heavy}} \frac{ds}{s^{k+1}} \text{Im} \mathcal{M}_\psi(s) \qquad (k > 1)$$

$$\text{Im} \mathcal{M}_\psi(s) = \sum_J |c_J|^2 P_J(1 - 2p^2/s)_\psi$$

(Im M = sum of Legendre's with positive coefficients)

positive

(low-energy couplings) = (sums of high-energy unknowns)

Warm-up: non-gravitational real scalar

Want to study effects of unknown UV on IR, so start with IR as simple as possible:
assume weak coupling for $s, t, u \ll M^2$, and $m_\phi \ll M$ so effectively massless.

[SCH& van Duong '20]

$$\mathcal{L}_{\text{low}} = -\frac{1}{2}(\partial_\mu\phi)^2 - \frac{g}{3!}\phi^3 - \frac{\lambda}{4!}\phi^4 + \frac{g_2}{2}\left[(\partial_\mu\phi)^2\right]^2 + \frac{g_3}{3}(\partial_\mu\partial_\nu\phi)^2(\partial_\sigma\phi)^2 + 4g_4\left[(\partial_\mu\partial_\nu\phi)^2\right]^2 + \cdots$$

$$\updownarrow$$

$$\mathcal{M}_{\text{low}}(s,t) = -g^2\left[\frac{1}{s} + \frac{1}{t} + \frac{1}{u}\right] - \lambda + g_2(s^2+t^2+u^2) + g_3(stu) + g_4(s^2+t^2+u^2)^2 + g_5(s^2+t^2+u^2)(stu) + \ldots$$

Goal: bound higher-derivative coefficients g_k

…assuming that unknown high-energy physics respects relativity and QM

($\Rightarrow$ amplitude stays analytic and bounded for $|s| > M^2$)

First few sum rules: (k=2, 4, …)

Low energies $\Leftrightarrow$ High energies

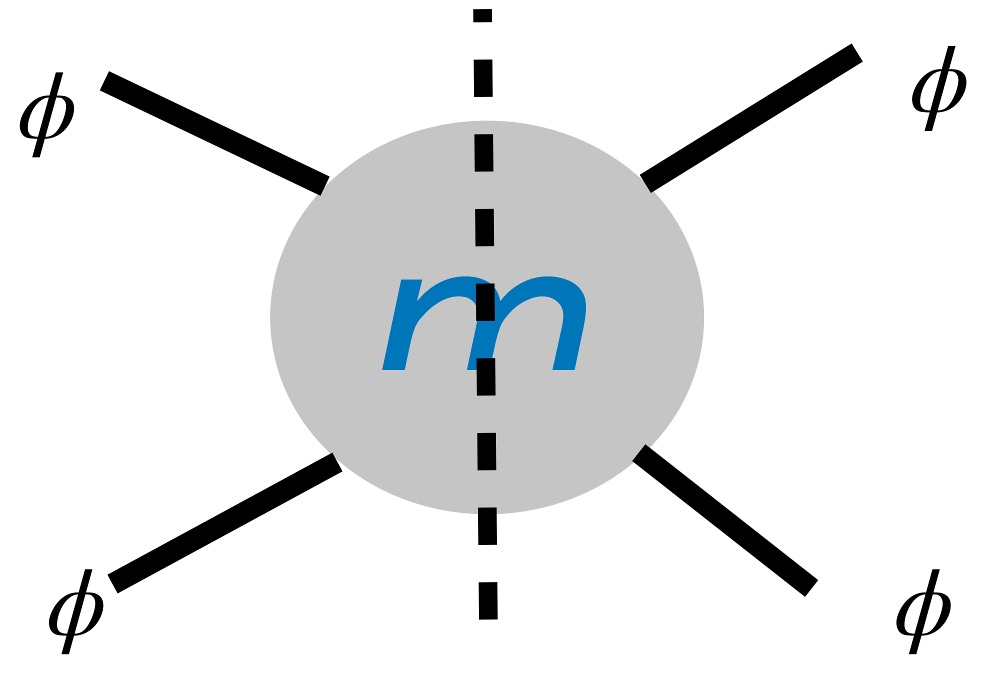
$$\oint_{\text{EFT}} \frac{ds\, M(s,t)}{s^{k+1}} = \int_{\text{heavy}} \frac{ds}{s^{k+1}} \mathrm{Im} M(s,t)$$

[to be integrated against $\psi(-t)$]

$$B_2: \quad 2g_2 - g_3 t + 8 g_4 t^2 + \dots = \left\langle \frac{\left(2m^2+t\right) \mathcal{P}_J\left(1+\frac{2t}{m^2}\right)}{m^2\left(m^2+t\right)^2} \right\rangle$$

$$B_4: \quad 4g_4 + \dots = \left\langle \frac{\left(2m^2+t\right) \mathcal{P}_J\left(1+\frac{2t}{m^2}\right)}{m^4\left(m^2+t\right)^3} \right\rangle$$

$\langle \cdots \rangle$ = sum over heavy states of energy $m > M$ and spin J, weighted by unknown $|c_J(m)|^2$



Let's get intuition by expanding around forward scattering: [! needs extra assumptions !]

$$g_2 = \left\langle \frac{1}{m^4} \right\rangle, \qquad g_3 = \left\langle \frac{3 - \frac{4}{d-2}\mathcal{J}^2}{m^6} \right\rangle \qquad g_4 = \left\langle \frac{1}{2m^8} \right\rangle_{m \geq M}$$

clearly: $g_2 \geq 0$ [since all unknowns in $\langle \ldots \rangle$ are positive]

Already excludes some interesting EFTs ("Galileons")!

[Adams,Arkani-Hamed,Dubovsky,Nicolis&Rattazzi '06]

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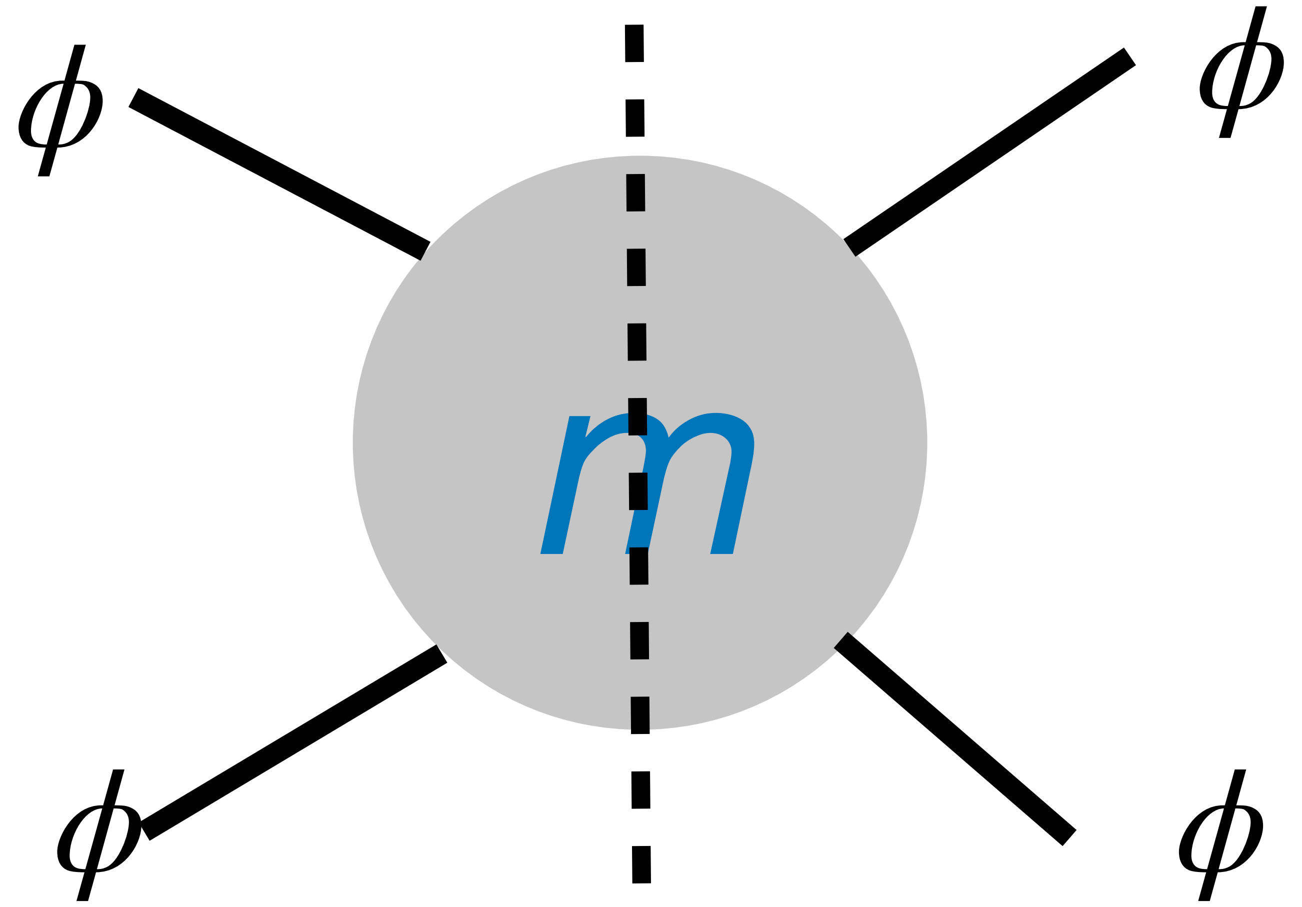
Clearly also: $g_3 \leq \frac{3g_2}{M^2}$, $0 \leq g_4 \leq \frac{g_2}{2M^4}$ [since $m \geq M$ in the average]

but how about a large negative g_3?

"null constraints" from IR crossing: $0 = \left\langle \frac{\mathcal{J}^2(2\mathcal{J}^2 - 5d + 4)}{m^8} \right\rangle$



An IR constraint on UV spectral density! (light-light-heavy couplings)

$$\left\langle \frac{1}{m^4} \underbrace{\frac{J^2}{m^2}}_{\sim b^2} \right\rangle_{m \geq M} \leq \frac{\#}{m^2} \left\langle \frac{1}{m^4} \right\rangle_{m \geq M}$$

[Tolley, Wang& Zhou '20]
[SCH& van Duong '20]

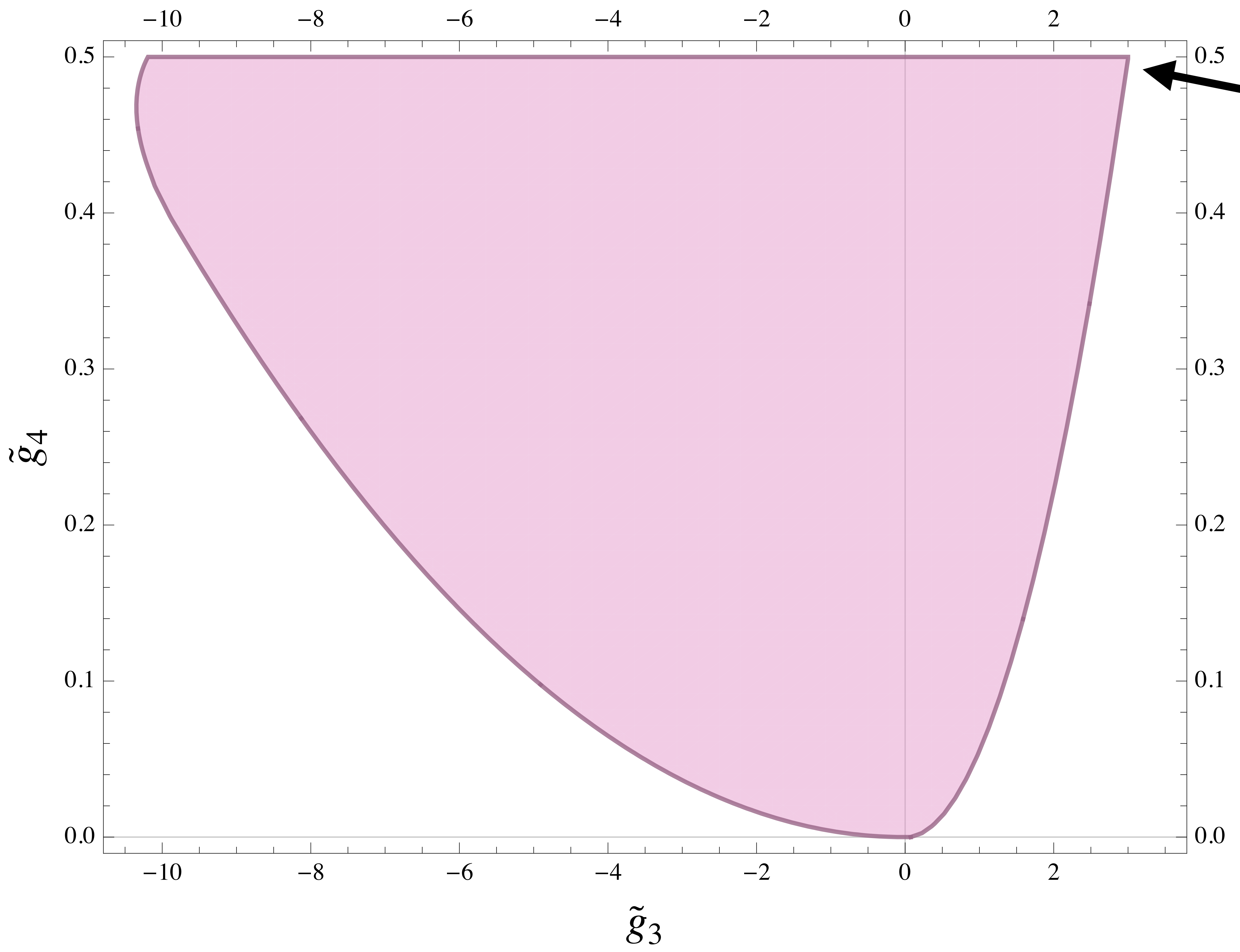
$\Rightarrow$ As far as sum rules are concerned,

heavy states with large spin (large b) can't couple strongly

(ie. large black holes, long strings, etc, can never dominate sum rules)

EFT dimensional analysis is a theorem

$\tilde{g}_k \equiv g_k M^{\#} / g_2$



Extremal EFTs take simple form:

$$\mathcal{M}_{\text{pole}} = \frac{g^2}{M^2 - s} + \text{perms}$$
$$= \frac{g^2}{M^2}\left(1 + \frac{s}{M^2} + \frac{s^2}{M^4} + \dots\right) + \text{perms}$$

Causality+Unitarity ⇒
UV states of mass> M cannot lead to
s^3 coefficient larger than s^2 coefficient$/M^2$.

[Tolley, Wang& Zhou '20]
[SCH& van Duong '20]
[Arkani-Hamed, Huang& Huang '20, …

- Modern approach: crossing-symmetric dispersion relations
  (does not require forward limits; stable against loops, …)

[Sinha& Zahed '20; see Pasiecznik '25 or my TASI 2025 lectures for more references]

What's special about gravity?

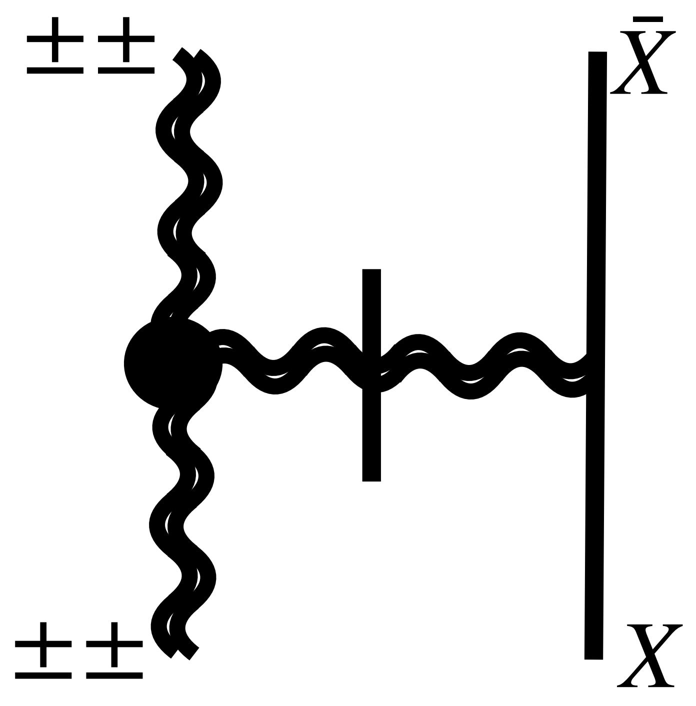
Causality and gravity: some known results

1) To leading power at long distances, any Lorentz-invariant S-matrix of a massless spin-2 particle must reproduce GR

[Weinberg]

2) Corrections: consider high-energy, large impact-parameter scattering.

If $M(s,b)$ exponentiates when $s \gg M_{\rm pl}^2$, eigenvalues=time delay/advance.

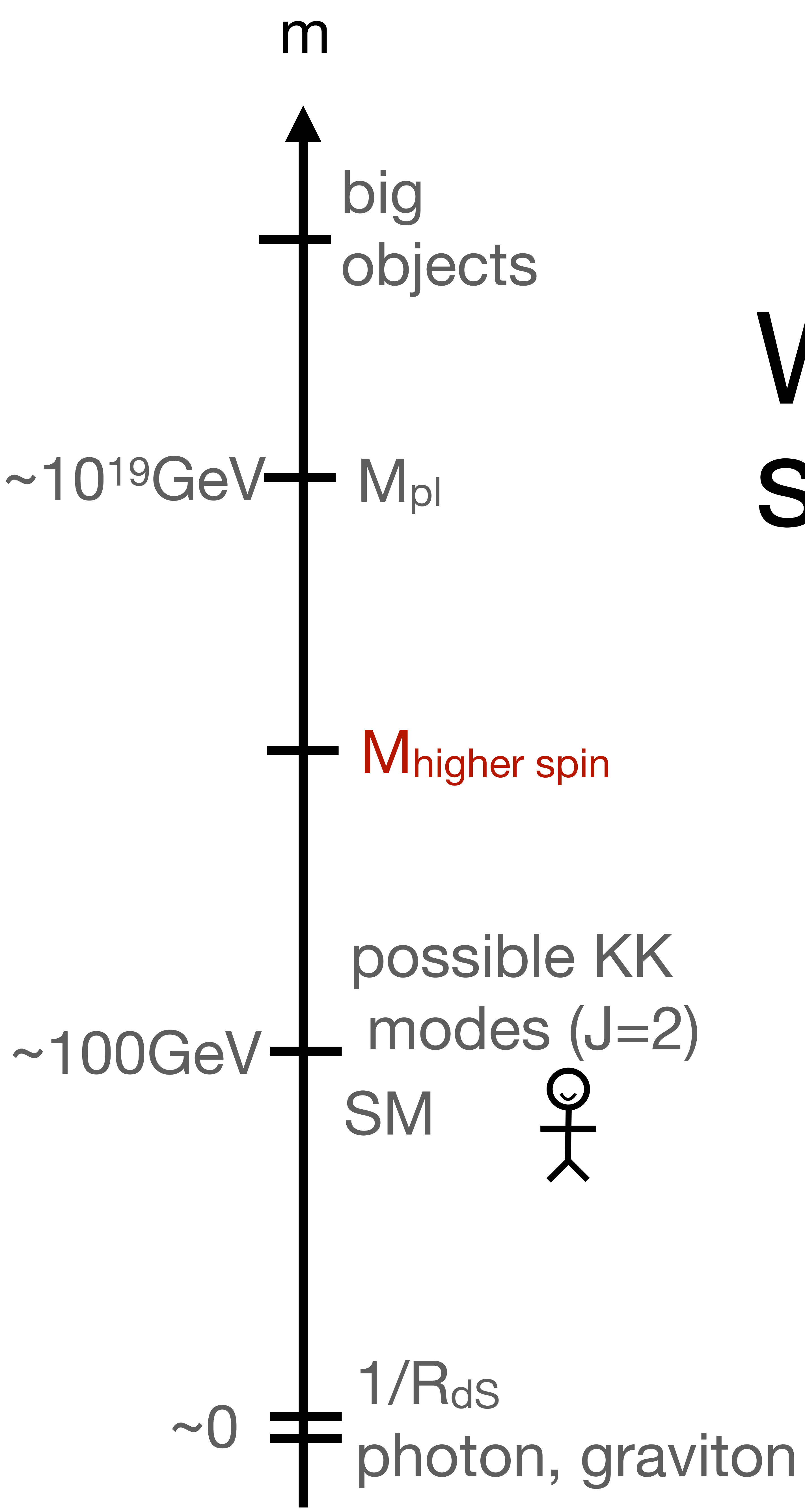


$$M(s,b) \sim 8\pi G s \begin{pmatrix} \log(1/b\mu_{\rm IR}) & \frac{g_3}{b^4} \\ \frac{g_3}{b^4} & \log(1/b\mu_{\rm IR}) \end{pmatrix} \geq 0 \qquad \Rightarrow \qquad |g_3| < \frac{\log(\cdots)}{b_{\rm min}^4}$$

[Camanho,Edelstein,Maldacena& Zhiboedov '14]

$\Rightarrow$ Classical theory with action $(R + g_3 {\rm Riem}^3)$ only respects causality for $b \gtrsim |g_3|^{1/4}$!

$\Rightarrow$ UV completion requires exotic higher-spin particles ($J \geq 4$) with $M \lesssim 1/b_{\rm min}$



Our setup

We are interested in graviton-graviton scattering below $M_{\text{higher-spin}}$

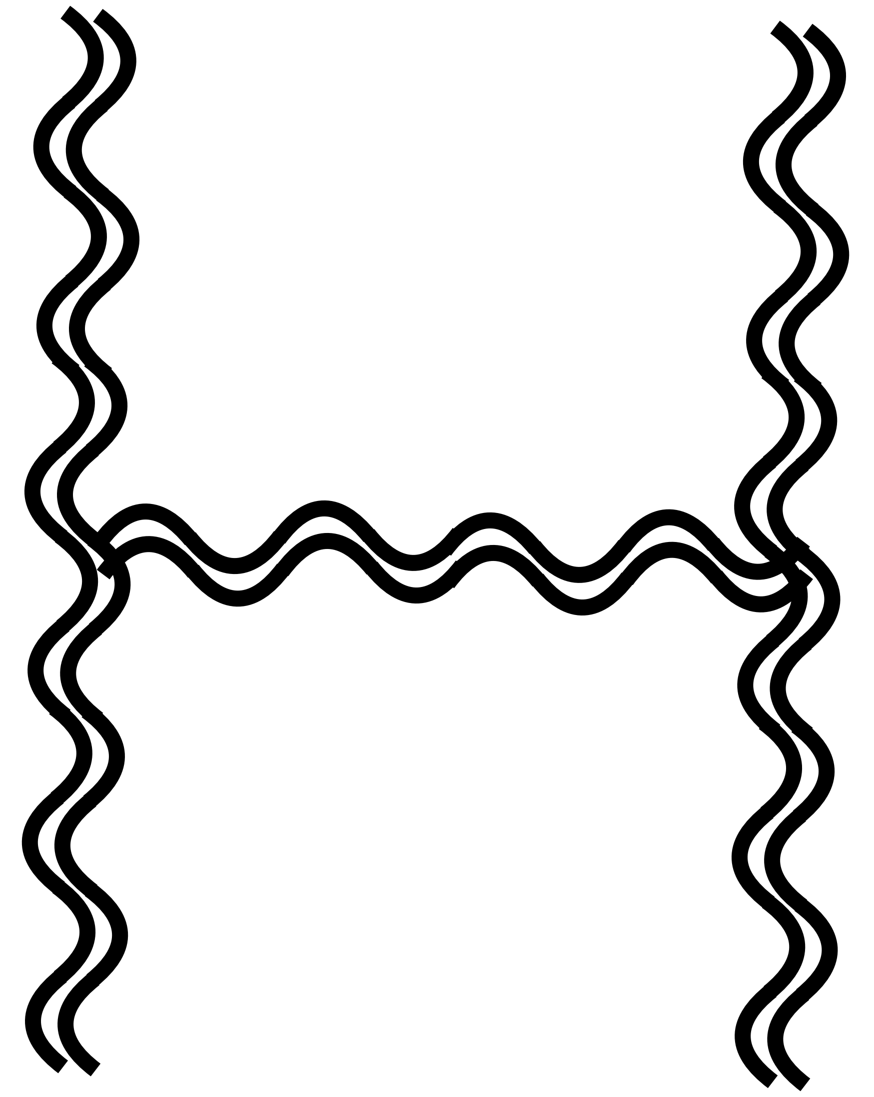
$$S = \frac{1}{16\pi G}\left(R + g_3 \mathrm{Riem}^3 + g_4 \mathrm{Riem}^4 + \ldots\right) + \mathrm{matter}$$

$\Rightarrow$ How large can #'s be ?

How small: [Guerrieri, Penedones& Vieira ‘21]

Our method: low energy graviton scattering

$M_{\text{higher-spin}} \ll M_{pl}$: neglect loops.



scalar with

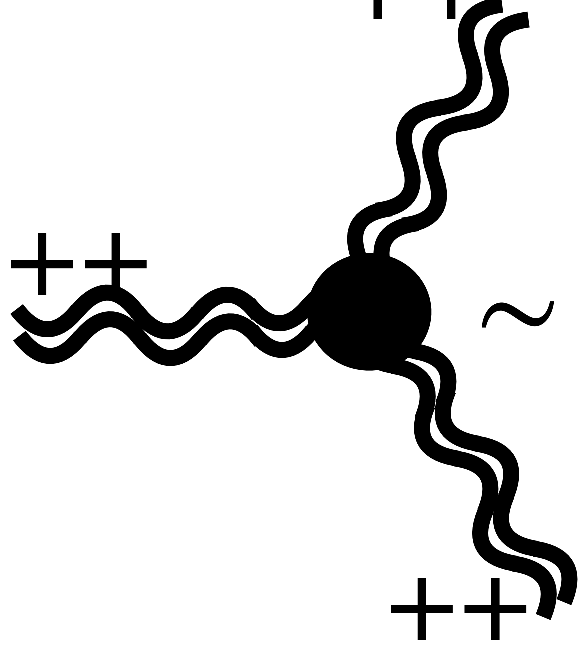
ϕRiem²/dCS

$$M^{+--+} = [14]^4\langle 23\rangle^4 \times 8\pi G\left[\frac{1}{stu} + \frac{|g_3|^2 su}{4t} + \frac{|g_s|^2}{-t} + g_4 + g_5 t + \ldots\right]$$

Einstein!

Riem³
at vertices

contacts: Riem⁴
and derivatives



$$\sim \sqrt{8\pi G}\, g_3 [12]^2[23]^2[13]^2$$

[SCH, Li, Parra Martinez, Simmons-Duffin `22]

We do not bound:

- $f(R)$ ($\simeq$ Einstein + scalar field : no imprint on graviton scattering)
- Any term with Ricci tensor/scalar: removable by field redefinition (no imprint)
- Scalar potentials (don't grow with energy)
- Torsion etc: treat as extra matter fields / non-minimal couplings to matter

We bound:

- Anything that affects graviton-graviton scattering thought experiments!

  + includes contacts $D^k \mathrm{Riem}^4$; scalar couplings $\phi \mathrm{Riem}^2$/dCS

  + use amplitude when it is small: no exponentiation needed

Dispersive sum rules for gravity: spin is good!

$$M^{+--+} = \overset{\propto\, t^4}{[14]^4\langle 23\rangle^4} \times 8\pi G \overset{f(s,t)}{\left[\frac{1}{stu} + \frac{|g_3|^2 su}{4t} + \frac{|g_s|^2}{-t} + g_4 + g_5 t + \ldots\right]}$$

- The prefactor grants superconvergent sum rules (no denominator):

$$B_2(u) : 0 = \oint_{s=\infty} (s-t)ds[f(s,t) + f(t,s)]\,, \qquad B_3(u) : 0 = \oint_{s=\infty} ds[f(s,t) - f(t,s)]$$

automatically kills all contacts!

- **Superconvergent** sum rules:

$$8\pi G_N\left[\frac{1}{p^2}+|g_s|^2p^2+|g_3|^2p^6\right]=\int_{\substack{m^2>M^2_{\text{higher-spin}}\,(J\geq 3)\\ m>0\text{ for }J\leq 2}}\frac{dm^2}{m^8}(2m^2-p^2)\sum_J\left[|c^J_{++}(m)|^2d^J_{++}(1+\frac{2p^2}{m^2})+|c^J_{+-}(m)|^2(\ldots)\right]$$

exact! (neglecting loops $\sim 1/M^4_{\text{pl}}$)

- Schematically, for any $p \leq M_{\text{higher-spin}}$:

$$\frac{G_N}{p^2}+\ldots=\sum_{J,m\geq M_{\text{higher-spin}}}c^2_{J,m}P_J(1-2p^2/m^2),$$

$\Rightarrow G_N > 0$
Gravity is attractive!!!!!

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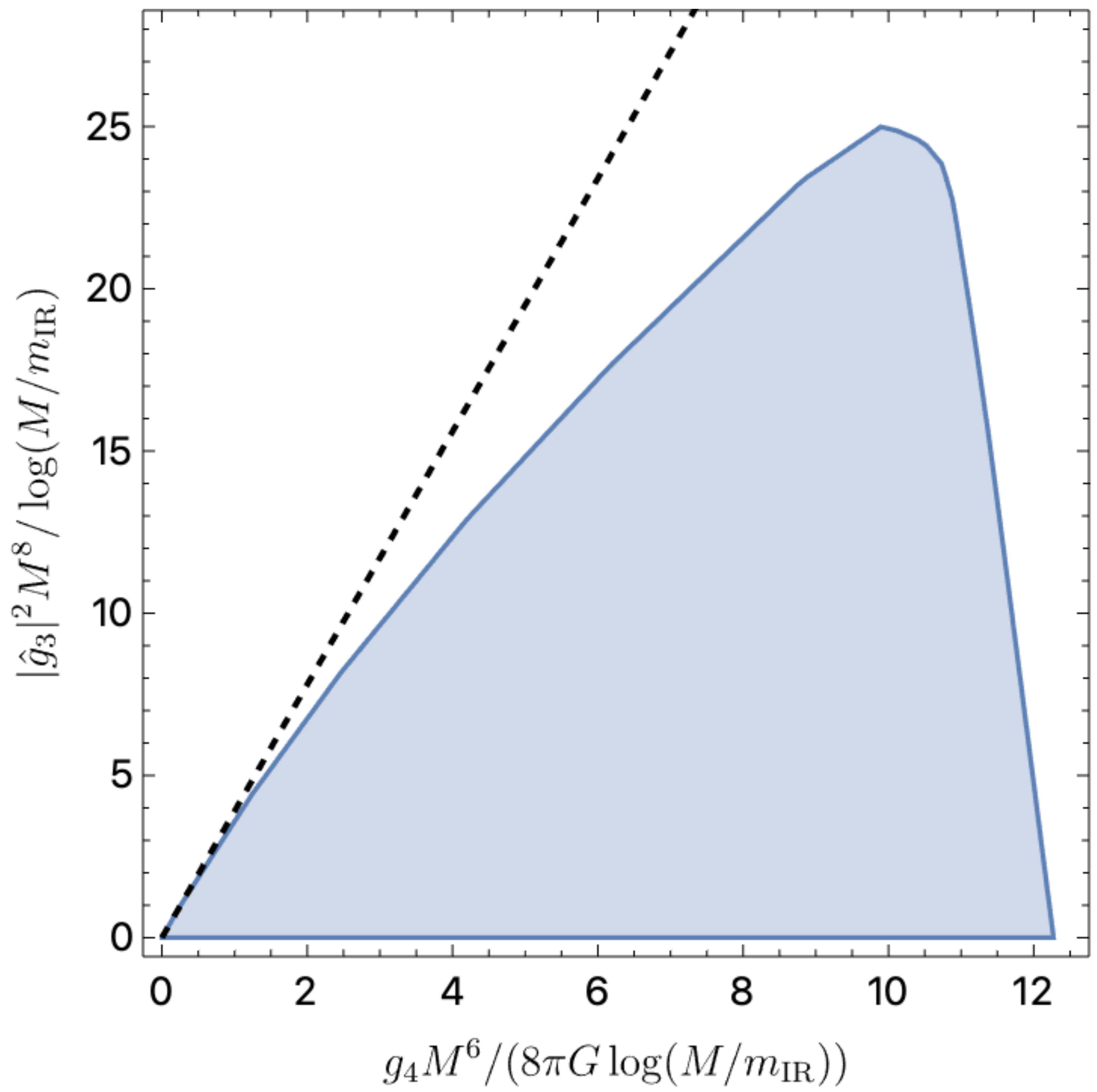
$\Rightarrow G_N>0$

Gravity is attractive!!!!!

$$G_N|g_s|^2=\sum_{J,m\geq M_{\text{higher-spin}}}\frac{\text{similar}}{m^4}$$

Every other interactions involving gravitons is dominated by $G_N/m^{\#}$!

Riem³ and Riem⁴ can never dominate over R within the regime of validity of a gravity EFT



$$S = \frac{1}{16\pi G}\int\left(R + \frac{\tilde{g}_3\ \mathrm{Riem}^3}{M^4_{\mathrm{higher-spin}}} + \frac{\tilde{g}_4\ \mathrm{Riem}^4}{M^6_{\mathrm{higher-spin}}} + \ldots\right)$$

rescaled $\tilde{g}$'s can't exceed O(1) without violating causality

[SCH, Li, Parra Martinez, Simmons-Duffin \`22]

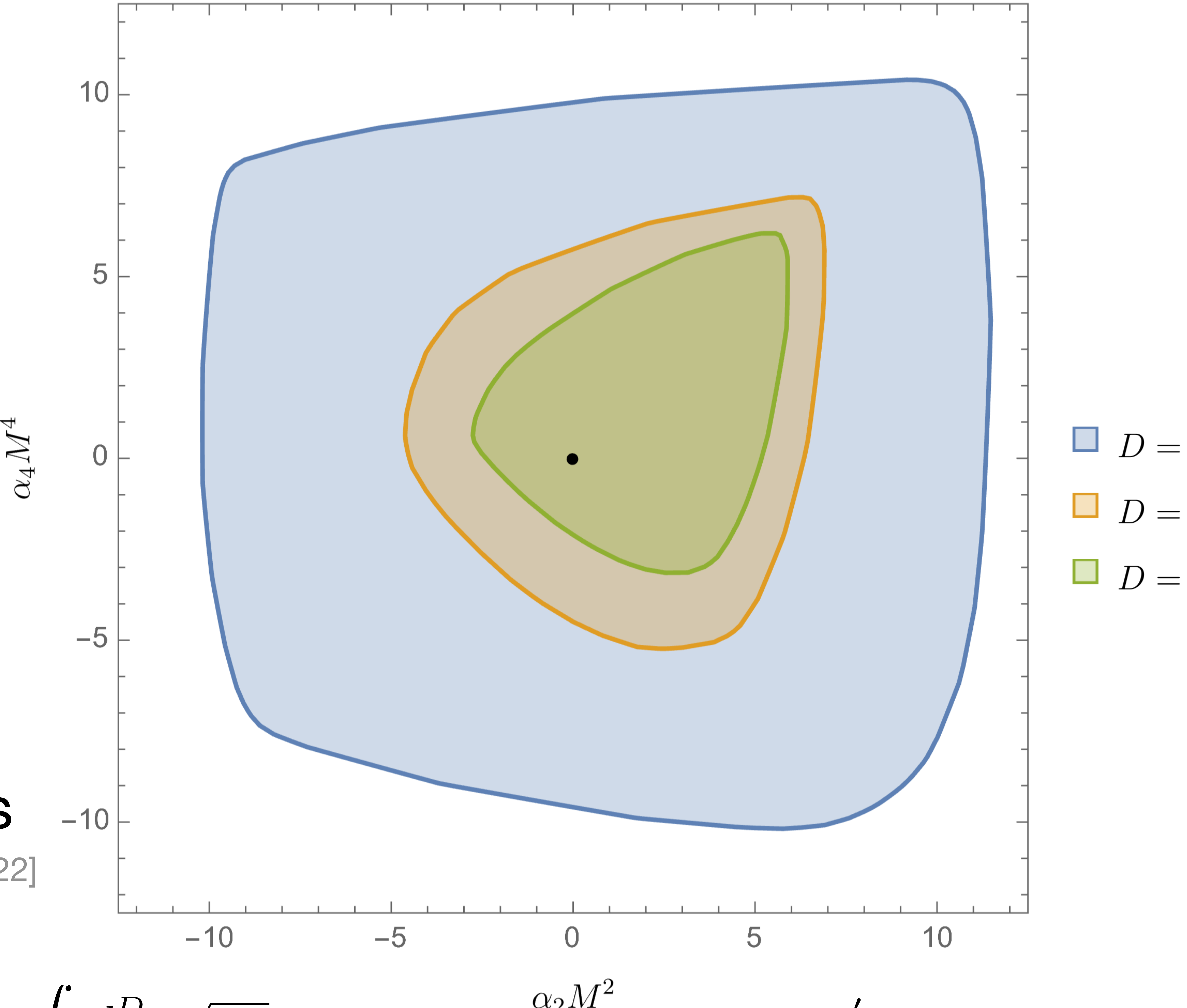
Comments on IR logs

Due to IR-divergent Shapiro time delay in GR, all bounds in D=3+1 involve $G_N \log(M/\mu_{\rm IR})$

- Cutoff can be related to resolution and size of experiment
  [Bellazzini, Berman, Isabella, Riva, Romano & Sciotti '25]

- In AdS$_4$, CFT bounds have $G_N \log(MR_{\rm AdS})$
  [SCH, Mazac, Rastelli& Simmons-Duffin `21]

- Bounds in $D \geq 5$ are finite an unambiguous
  [SCH, Li, Parra Martinez, Simmons-Duffin `22]



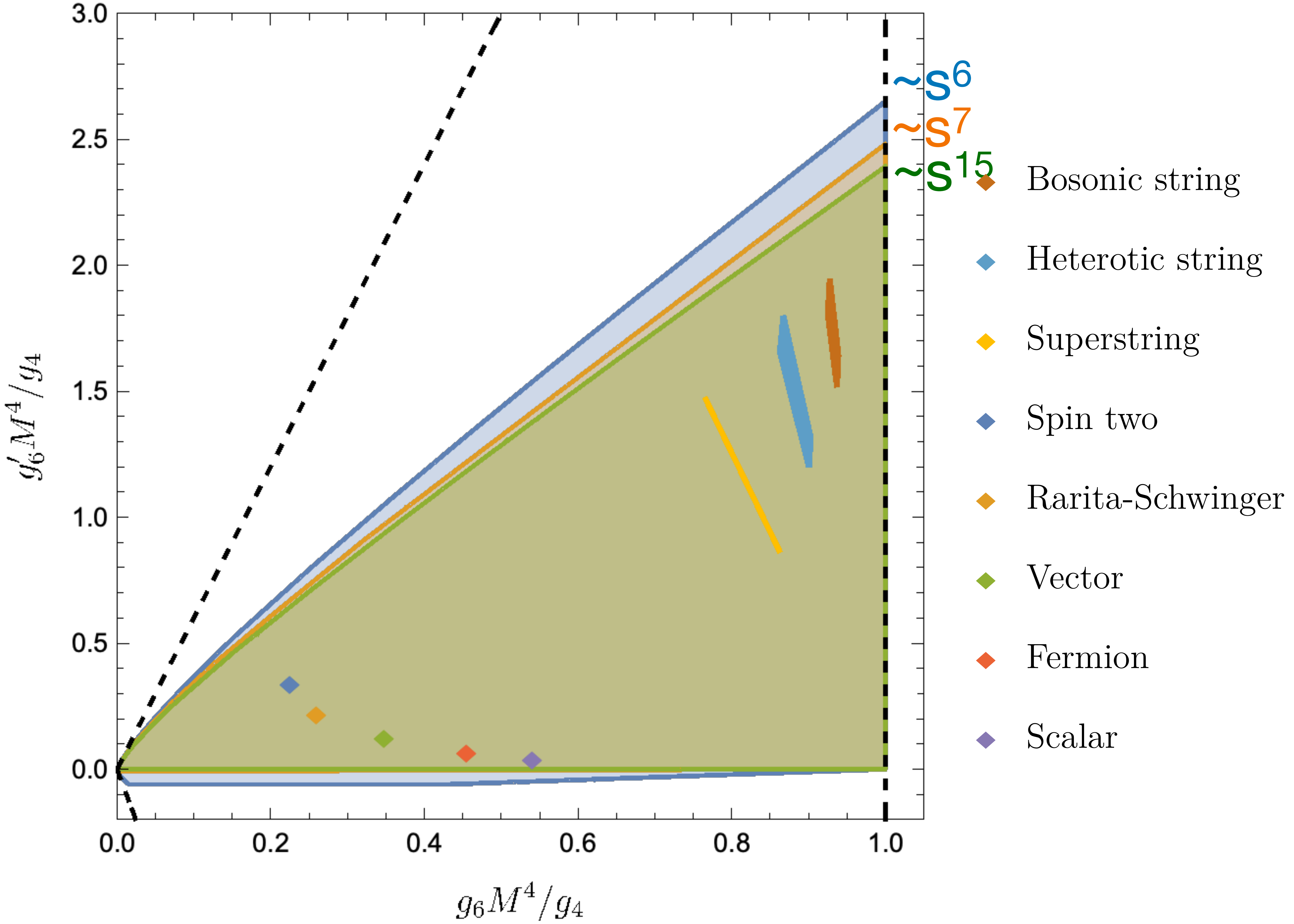
$$\left|\frac{a-c}{c}\right| \leq \frac{23}{\Delta_{\rm gap}^2} + \mathcal{O}(1/\Delta_{\rm gap}^4)$$

$$S = \int \frac{d^D x\sqrt{-g}}{16\pi G}\left(R + \frac{\alpha_2}{4}C^2 + \frac{\alpha_4}{12}C^3 + \frac{\alpha_4'}{6}C'^3 + \dots\right)$$

More derivatives: $D^4 Riem^4$ normalized by $Riem^4$

[SCH, Li, Parra Martinez, Simmons-Duffin `22]

All higher-derivatives coefficients are bounded by $g_{\text{Riem}^4}/M^{\#}_{\text{higher-spin}}$!



Dashed lines (slopes -90/11 and +6) coincide with homogeneous bounds from [Bern, Kosmopoulos& Zhiboedov '21]

What remains consistent with causality+unitarity?

$$S = \int d^4x \left(\frac{R}{16\pi G} - \frac{1}{2} g_{ij}(\phi) D_\mu \phi^i D^\mu \phi^j + \text{other matter terms} \right)$$

- Gravity must couple minimally to itself and matter (partial results)

($\alpha\phi\mathrm{Riem}^2$ needs $|\alpha| \leq M_{\mathrm{pl}}/M^2_{\mathrm{higher-spin}}, \ldots$)

- Dim 5,6,7 matter self-interactions largely unconstrained (no ~s^2 growth)

Other exclusions:

- An isolated massive spin 2 graviton is not positive!

  dGRT gravity already known to have cutoff $\Lambda \leq (m_g^2 M_{\mathrm{pl}})^{1/3}$. Now $\Lambda \leq (\mathrm{few}) m_g$

[Bellazzini, Isabella, Ricossa& Riva ’23]

- A light spin-3/2 particle implies gravity and some “near”-supersymmetry

[Bellazzini, Pomarol, Romano & Sciotti ’25]

Summary

- General principles of relativity and quantum mechanics constrain size& sign of EFT coefficients
- Graviton scattering below $M_{\text{higher-spin}}$ can't significantly differ from GR

Some future questions

- What would be the implications of higher-spin exotics $(J > 2)$?

  - Could they decouple from SM and have ~km wavelength?
    …or imply departure from $1/r^2$ force law in tabletop experiments?
    …or already constrained to $< 1/(10\mathrm{TeV}) \sim 10^{-20}\mathrm{m}$ by colliders?
  - Conjectured to signal a universal cutoff of field theory
  - Could they be qualitatively different from string theory? [SCH+Li '24]

- Massive spin-2: can we constrain possible compactifications? [in progress]

- Can similar techniques constrain correlation functions of stress tensors and currents in arbitrary (non-conformal) CFTs?