Neutrino Theory 1967-2017

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Contents

Neutrino Mass in Perspective

Weinberg (1967): Minimal $SU(2)_L \times U(1)_Y$ Standard Model (SM) with one Higgs doublet $(\phi^+, \phi^0) \sim (2, 1/2)$ and lepton content $(\nu_e, e)_L, (\nu_\mu, \mu)_L \sim (2, -1/2)$, $e_R, \mu_R \sim (1, -1)$.
As ϕ^0 acquires a nonzero vacuum expectation value v, the charged leptons acquire masses through $\bar{e}_L e_R \phi^0$ and $\bar{\mu}_L \mu_R \phi^0$. Neutrinos are massless and their interactions are limited to
$(g/\sqrt{2})\bar{\nu}_L \gamma^\mu l_L W^+_\mu + H.c. + (g/2\cos\theta_W)\bar{\nu}_L \gamma^\mu \nu_L Z_\mu$.
New particles are $W^\pm, Z^0$ and $h = \sqrt{2}(\phi^0 - v)$.

Weinberg (1979): If physics beyond the SM occur at mass scales much greater than the electroweak breaking scale v, there is a unique dimension-five operator for Majorana neutrino mass:

$$\mathcal{L}_5 = \frac{f_{\alpha\beta}}{2\Lambda}(\nu_\alpha \phi^0 - l_\alpha \phi^+)(\nu_\beta \phi^0 - l_\beta \phi^+)$$

Hence $\mathcal{M}_\nu = f_{\alpha\beta} v^2/\Lambda$. This shows that seesaw is always obtained for $v << \Lambda$.

Exception: If new physics occurs at scales lower than v, then other possibilities exist.

Early Specific Ideas

(1979) Add N_R with m_N very large, then

$$\begin{pmatrix} 0 & m_D \\ m_D & m_N \end{pmatrix} \Rightarrow m_\nu \simeq \frac{-m_D^2}{m_N}.$$

This is the famous original seesaw mechanism (coined by Yanagida), and it implies only the interaction $f \bar{N}_R \nu_L \phi^0$ which generates $m_D = fv$.

(1980) Add $(\xi^{++}, \xi^{+}, \xi^{0})$ with $\langle \xi^0 \rangle = u$, then $(f/2)\xi^0 \nu\nu \Rightarrow m_\nu = fu$.

This scalar triplet mechanism implies new scalar particles with gauge interactions as well as

$$V = m^2\Phi^\dagger\Phi + M^2\xi^\dagger\xi + \frac{1}{2}\lambda_1(\Phi^\dagger\Phi)^2 + \frac{1}{2}\lambda_2(\xi^\dagger\xi)^2$$

$$+\lambda_3|2\xi^{++}\xi^0 - \xi^+\xi^+|^2 + \lambda_4(\Phi^\dagger\Phi)(\xi^\dagger\xi)$$

$$+\frac{1}{2}\lambda_5[|\sqrt{2}\xi^{++}\phi^- + \xi^+\bar{\phi}^0|^2 + |\xi^+\phi^- + \sqrt{2}\xi^0\bar{\phi}^0|^2]$$

$$+\mu(\bar{\xi}^0\phi^0\phi^0 + \sqrt{2}\xi^-\phi^0\phi^+ + \xi^{--}\phi^+\phi^+) + H.c.$$

If $\mu = 0$, then ξ may be assigned $L = -2$, and $u \neq 0$ breaks it spontaneously, resulting in a massless Goldstone boson (triplet majoron), ruled out by Z decay.

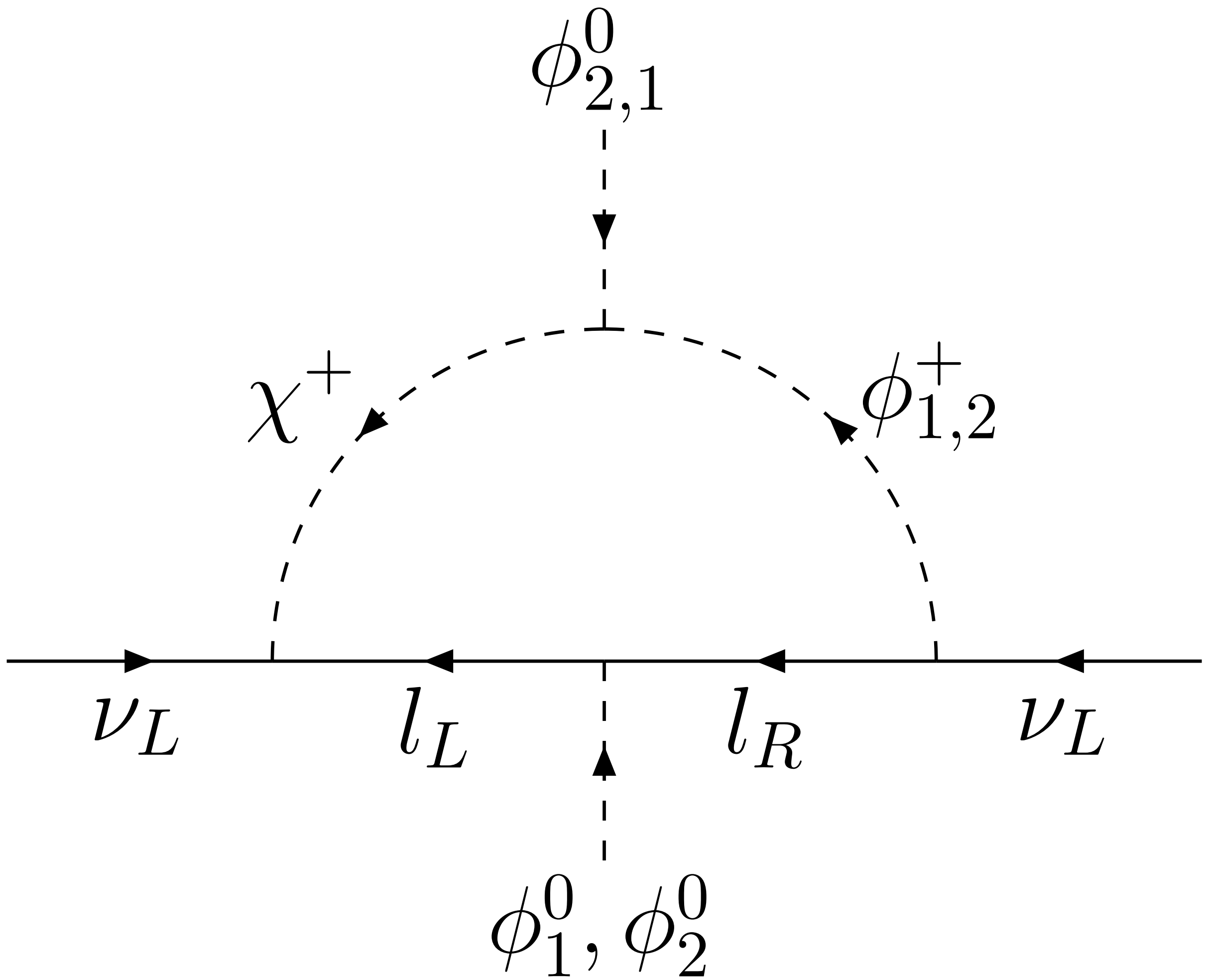
The minimum of V is given by

$$m^2 + \lambda_1 v^2 + (\lambda_4 + \lambda_5) u^2 + 2\mu u = 0,$$

$$u[M^2 + \lambda_2 u^2 + (\lambda_4 + \lambda_5) v^2] + \mu v^2 = 0.$$

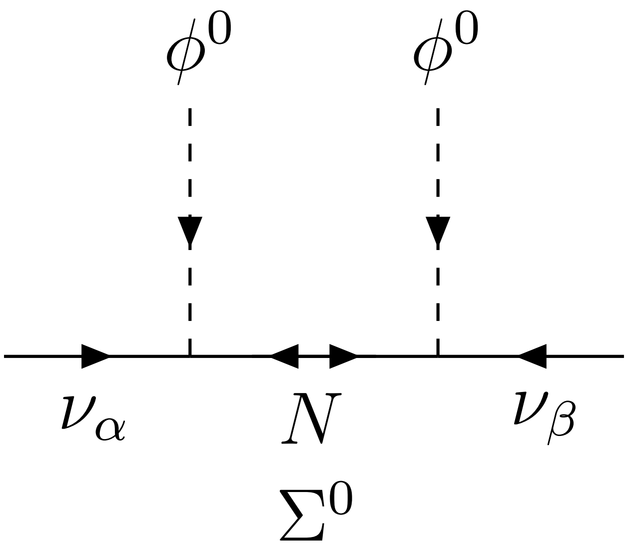
For $\mu \neq 0$, $u \simeq -\mu v^2/[M^2 + (\lambda_4 + \lambda_5) v^2]$, where $v^2 \simeq -m^2/\lambda_1$. Note that this is also a seesaw formula for $v^2 << M^2$.

(1980): Add charged singlet scalar χ^+ and second scalar doublet (ϕ_2^+, ϕ_2^0), then a one-loop m_ν is generated. This also implies new gauge and other interactions.

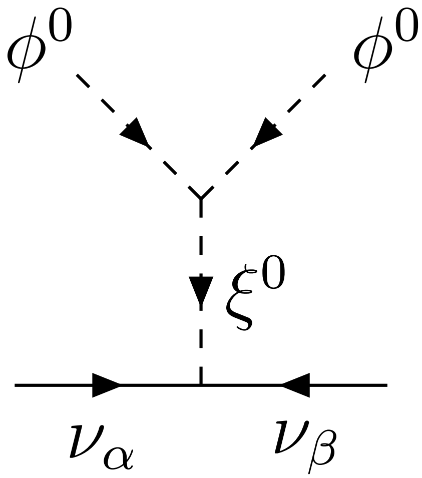

$\phi^0_{2,1}$
χ^+
$\phi^+_{1,2}$
ν_L
l_L
l_R
ν_L
ϕ^0_1, ϕ^0_2

(1989): Add $(\Sigma^+,\Sigma^0,\Sigma^-)_R$, then $\bar{\Sigma}^0_R \nu_L \phi^0 \Rightarrow m_D$, and $m_\nu \simeq -m_D^2/m_\Sigma$. Gauge interactions are implied.

(1998): [Ma: PRL 81, 1171 (1998)]
$\mathcal{L}_5$ has three and only three tree-level realizations:
(I) fermion singlet N (1979),
(II) scalar triplet (ξ^{++},ξ^+,ξ^0) (1980),
(III) fermion triplet $(\Sigma^+,\Sigma^0,\Sigma^-)$ (1989);
and three generic one-loop realizations:
(R1) (Zee, 1980), (R2) (Ma, 2006), (R3) (Fraser/Ma/Popov, 2014). The nomenclature of Type I, II, III seesaw first appeared there and is now well established.



ϕ^0
ϕ^0
ν_α
N
ν_β
Σ^0



ϕ^0
ϕ^0
ξ^0
ν_α
ν_β

(I) $(\phi^0\nu_i - \phi^+ l_i)(\phi^0\nu_j - \phi^+ l_j)$,

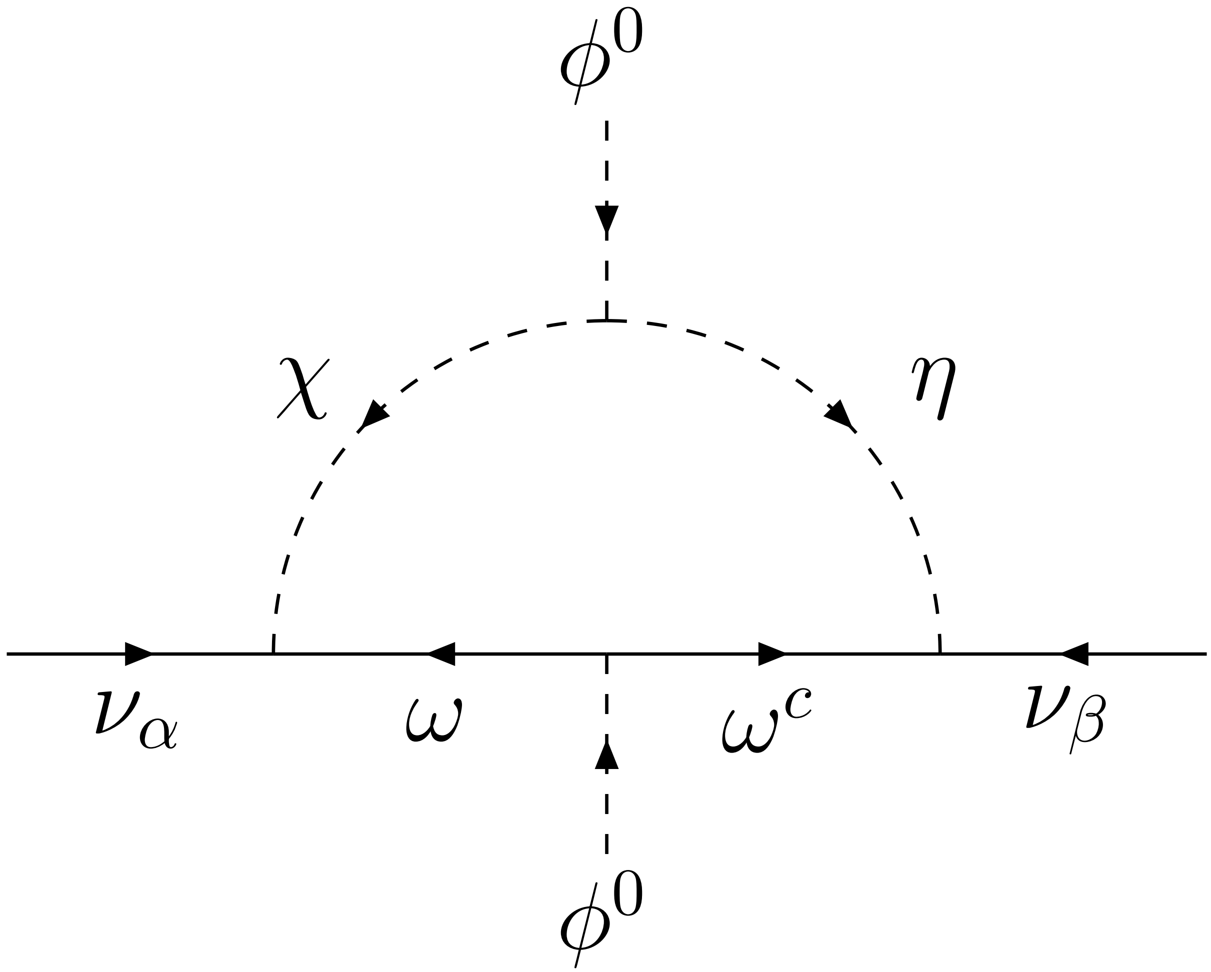
(II) $\phi^0\phi^0\nu_i\nu_j - \phi^+\phi^0(\nu_i l_j + l_i\nu_j) + \phi^+\phi^+ l_i l_j$,

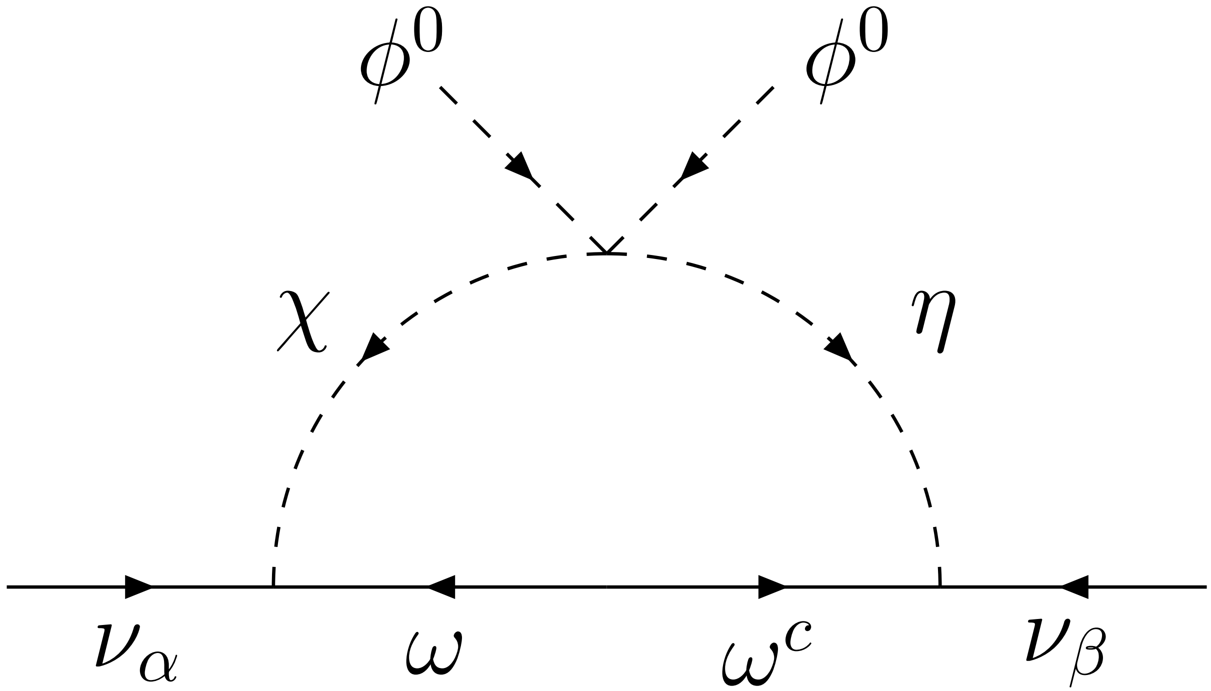
(III) $(\phi^0\nu_i + \phi^+ l_i)(\phi^0\nu_j + \phi^+ l_j) - 2\phi^+\nu_i\phi^0 l_j - 2\phi^0 l_i\phi^+\nu_j$.

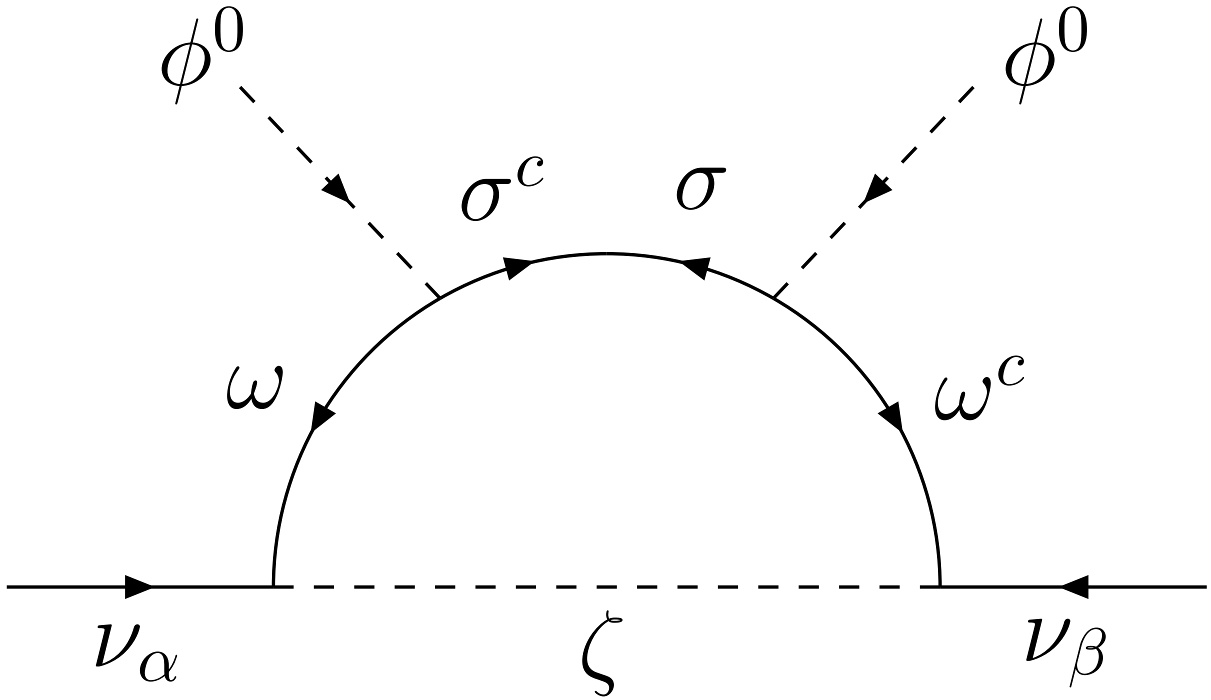
Radiative mechanism: The two external neutrino lines are connected in one loop by an internal fermion line and an internal scalar line.

(R1) The two external Higgs lines are attached one to the scalar line and one to the fermion line.

(R2, R3) The two Higgs lines are both attached to the (scalar, fermion) line.


ϕ^0
χ
η
ν_α
ω
ω^c
ν_β
ϕ^0


ϕ^0
ϕ^0
χ
η
ν_α
ω
ω^c
ν_β


ϕ^0
ϕ^0
σ^c
σ
ω
ω^c
ν_α
ζ
ν_β

Super-K (June, 1998): [Neutrino 98 Conference, Japan] Atmospheric neutrino oscillations established.

Headline news around the world: Neutrinos Have Mass!!

SNO (2002): Solar neutrino oscillations established.

Absent of other information, how neutrinos get their mass is still unknown. In fact, we still do not know if its mass is Dirac or Majorana, without a positive signal from neutrinoless double beta decay.

(2012): The 125 GeV particle was discovered at CERN, but no other new particle since then. It is presumably the SM Higgs boson h, with important implications.

New ideas on Lepton Number

Lepton number is usually thought of as being an integer L or a parity $(-1)^L$. In the latter case, neutrinos are Majorana, which is the default option. In the former case, they are Dirac, and in the persisting nonobservation of neutrinoless double beta decay, there is a theoretical resurgence of interest in them.

What is new in the last few years is the realization that lepton number may be based on Z_n. There are already explicit examples of Z_3 and Z_4 models.

The former allows long-lived Z_3 dark matter decaying only to neutrinos.

The latter predicts the unusual phenomenon of neutrinoless quadruple beta decay.

A recent extension of an old idea is to connect $U(1)_{PQ}$ to $U(1)_L$ with the addition of color triplet or color octet fermions. This allows the QCD axion to be identified with a would-be singlet Majoron.

It is also possible to have the light mediator of self-interacting dark matter decaying to neutrinos, because it is a would-be singlet-triplet Majoron.

Radiative Seesaw from Dark Matter

Deshpande/Ma(1978): Add to the SM a second scalar doublet (η^+, η^0) which is odd under a new exactly conserved Z_2 discrete symmetry, then η^0_R or η^0_I is absolutely stable. This simple idea lay dormant for almost thirty years until Ma, Phys. Rev. D 73, 077301 (2006). It was then studied seriously in
Barbieri/Hall/Rychkov(2006),
Lopez Honorez/Nezri/Oliver/Tytgat(2007),
Gustafsson/Lundstrom/Bergstrom/Edsjo(2007), and
Cao/Ma/Rajasekaran, Phys. Rev. D 76, 095011 (2007).

Radiative Neutrino Mass:

Zee(1980): (R1)

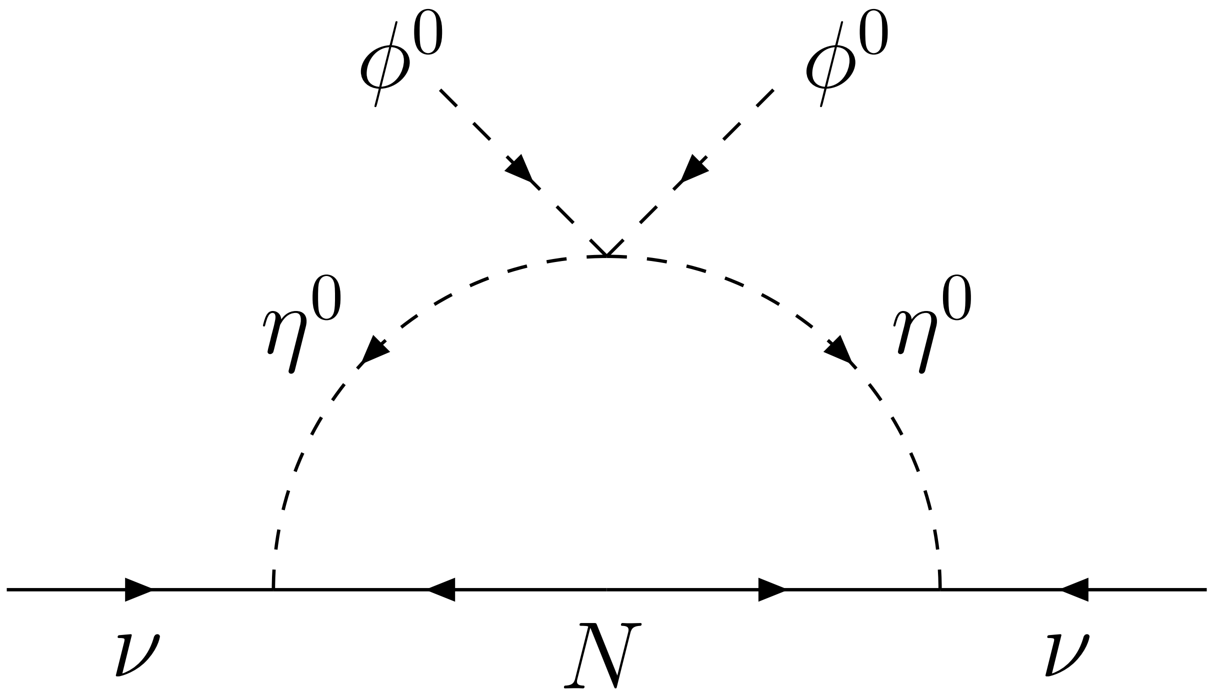
$\omega = (\nu, l), \omega^c = l^c$, $\chi = \chi^+, \eta = (\phi^+_{1,2}, \phi^0_{1,2}), \langle \phi^0_{1,2} \rangle \neq 0$.

Ma(2006): (R2) [scotogenic = caused by darkness]

$\omega = \omega^c = N$ or Σ, $\chi = \eta = (\eta^+, \eta^0), \langle \eta^0 \rangle = 0$.

N or Σ interacts with ν, but they are not Dirac mass partners, because of the exactly conserved Z_2 symmetry, under which N or Σ and (η^+, η^0) are odd, and all SM particles are even. Using $f(x) = -\ln x/(1-x)$,

$$(\mathcal{M}_\nu)_{\alpha\beta} = \sum_i \frac{h_{\alpha i} h_{\beta i} M_i}{16\pi^2} [f(M_i^2/m_R^2) - f(M_i^2/m_I^2)].$$


ϕ^0
ϕ^0
η^0
η^0
ν
N
ν

Note that $(1/2)\lambda_5(\Phi^\dagger\eta)^2 + H.c.$ splits the mass of $\eta^0 = (\eta_R + i\eta_I)/\sqrt{2}$, so that $m_R^2 - m_I^2 = 2\lambda_5 v^2$, which makes m_ν finite.

It also solves the problem of the direct detection of $\eta_{R,I}$ through Z exchange with nuclei. Since Z_μ couples to $\eta_R\partial^\mu\eta_I - \eta_I\partial^\mu\eta_R$, a mass gap of just a few hundred keV is enough to forbid its elastic scattering in underground dark-matter search experiments using nucleus recoil.

However, the $h(\eta_R^2 + \eta_I^2)$ coupling remains. It will allow $\eta_{R,I}$ to be discovered in the next generation of detectors.

Neutrino Flavor Symmetry

Brief History of A_4: In 1978 (39 years ago), soon after the putative discovery of the third family of leptons and quarks, it was conjectured independently by Cabibbo and Wolfenstein:

$$U_{l\nu} = U_\omega = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix},$$

where $\omega = \exp(2\pi i/3) = -1/2 + i\sqrt{3}/2$. In the PDG convention, this implies $s_{23} = c_{23} = 1/\sqrt{2}$, $s_{12} = c_{12} = 1/\sqrt{2}$, $s_{13} = 1/\sqrt{3}$, $c_{13} = \sqrt{2/3}$, and $\delta = \pi/2$. If $\omega \leftrightarrow \omega^2$, then $\delta = -\pi/2$.

In 2001 (16 years ago), without knowing about Cabibbo and Wolfenstein, U_ω was discovered by Ma and Rajasekaran in the context of A_4.

This non-Abelian discrete symmetry has 12 elements and 4 irreducible representations: $\underline{1}, \underline{1}', \underline{1}'', \underline{3}$. Using

$$\underline{3} \times \underline{3} = \underline{1} + \underline{1}' + \underline{1}'' + \underline{3} + \underline{3}.$$

the following decompositions are obtained:

$$\begin{aligned} \underline{1} &= 11 + 22 + 33, \\ \underline{1}' &= 11 + \omega 22 + \omega^2 33, \\ \underline{1}'' &= 11 + \omega^2 22 + \omega 33. \end{aligned}$$

Let $(\nu, l)_i \sim \underline{3}$, $l_i^c \sim \underline{1}, \underline{1}', \underline{1}''$, and $\Phi_i \sim \underline{3}$, then

$$\mathcal{M}_l = \begin{pmatrix} f_e v_1^* & f_\mu v_1^* & f_\tau v_1^* \\ f_e v_2^* & f_\mu \omega v_2^* & f_\tau \omega^2 v_2^* \\ f_e v_3^* & f_\mu \omega^2 v_3^* & f_\tau \omega v_3^* \end{pmatrix}$$

$$= \begin{pmatrix} v_1^* & 0 & 0 \\ 0 & v_2^* & 0 \\ 0 & 0 & v_3^* \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} f_e & 0 & 0 \\ 0 & f_\mu & 0 \\ 0 & 0 & f_\tau \end{pmatrix}.$$

For $v_1 = v_2 = v_3$, a residual Z_3 symmetry exists with $U_\omega^\dagger$ as the link between $\mathcal{M}_l$ and $\mathcal{M}_\nu$.

For many years, theoretical effort was focused on obtaining a specific form of $\mathcal{M}_\nu$ so that tribimaximal neutrino mixing is realized:

$$U_{TBM} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} =$$

$$\frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & i \end{pmatrix}.$$

This means that

$$\mathcal{M}_\nu = \begin{pmatrix} m_2 & 0 & 0 \\ 0 & (m_1 - m_3)/2 & (m_1 + m_3)/2 \\ 0 & (m_1 + m_3)/2 & (m_1 - m_3)/2 \end{pmatrix}.$$

Pioneer A_4 papers: Ma/Rajasekaran(2001), Ma(2002), Babu/Ma/Valle(2003), Ma(2004), Altarelli/Feruglio(2005), Babu/He(2005).

This $\mathcal{M}_\nu$ is very hard to obtain in the context of a four-dimensional renormalizable field theory, because of the basic clash (or misalignment) of the residual symmetries (Z_3 for $\mathcal{M}_l$ and Z_2 for $\mathcal{M}_\nu$) [Lam]

On March 8, 2012, Daya Bay announced that θ_{13} had been measured at 8.8°, thus ending tribimaximal mixing.

The 2016 PDG values are: $\sin^2(\theta_{12}) = 0.304 \pm 0.014$,
$\Delta m^2_{21} = (7.53 \pm 0.18) \times 10^{-5}\ \mathrm{eV}^2$,
$\sin^2(\theta_{23}) = 0.51 \pm 0.05$,
$\Delta m^2_{32} = (2.44 \pm 0.06) \times 10^{-3}\ \mathrm{eV}^2$ (normal),
$\sin^2(\theta_{23}) = 0.50 \pm 0.05$,
$\Delta m^2_{32} = (2.51 \pm 0.06) \times 10^{-3}\ \mathrm{eV}^2$ (inverted),
$\sin^2(\theta_{13}) = (2.19 \pm 0.12) \times 10^{-2}$.

In retrospect, the $Z_3 - Z_2$ clash should have been a warning against tribimaximal mixing.

Special Form of $\mathcal{M}_\nu$: Ma(2002), Babu/Ma/Valle(2003), Grimus/Lavoura(2004):

A special form of the neutrino mass matrix (in the basis where the charged-lepton mass matrix is diagonal) was written down 15 years ago, i.e.

$$\mathcal{M}_\nu = \begin{pmatrix} A & C & C^* \\ C & D^* & B \\ C^* & B & D \end{pmatrix},$$

where A, B are real.

θ_{13} and θ_{12} are determined by $s_{13}/c_{13} = -D_I/\sqrt{2}C_R$, $s_{13}c_{13}/(c_{13}^2 - s_{13}^2) = \sqrt{2}C_I/(A - B + D_R)$, and

$$\frac{s_{12}c_{12}}{c_{12}^2 - s_{12}^2} = \frac{-\sqrt{2}(c_{13}^2 - s_{13}^2)C_R}{c_{13}[c_{13}^2(A - B - D_R) + 2s_{13}^2 D_R]}.$$

The three neutrino masses are determined by

$$m_2 + m_1 \simeq A + B + D_R + s_{13}^2(A - B + D_R),$$

$$(c_{12}^2 - s_{12}^2)(m_2 - m_1) \simeq -A + B + D_R - s_{13}^2(A - B + D_R),$$

$$m_3 \simeq -B + D_R + s_{13}^2(A - B + D_R).$$

This allows $\theta_{13} \neq 0$ and yet $\theta_{23} = \pi/4$ is maintained, together with the prediction that $\delta_{CP} = \pm\pi/2$. This cobimaximal pattern is protected by a symmetry, i.e. $e \to e$ and $\mu \leftrightarrow \tau$ exchange with CP conjugation. Present T2K data with input from reactor data indicate a preference for $\delta_{CP} = -\pi/2$.

Note that this special form predicts that $|U_{\mu i}| = |U_{\tau i}|$. This harkens back to the original U_ω of 1978, where indeed this is satisfied. It is strongly suggestive that U_ω itself must have something to do with the realization of this special form of $\mathcal{M}_\nu$.

Whereas tribimaximal mixing is dead, A_4 is not.
The simple yet crucial observation is that if $U_{l\nu} = U_\omega{}^\dagger \mathcal{O}$, where $\mathcal{O}$ is orthogonal, then $U_{2i}^* = U_{3i}$ for $i = 1, 2, 3$.
Compared this to the PDG form of $U_{l\nu}$, i.e.

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix},$$

it is obvious that after rotating the phases of the third column and the second and third rows, the two matrices are identical if and only if $s_{23} = c_{23}$ and $\cos\delta = 0$, i.e. $\theta_{23} = \pi/4$ and $\delta_{CP} = \pm\pi/2$.
[Fukuura/Miura/Takasugi/Yoshimura(2000)]

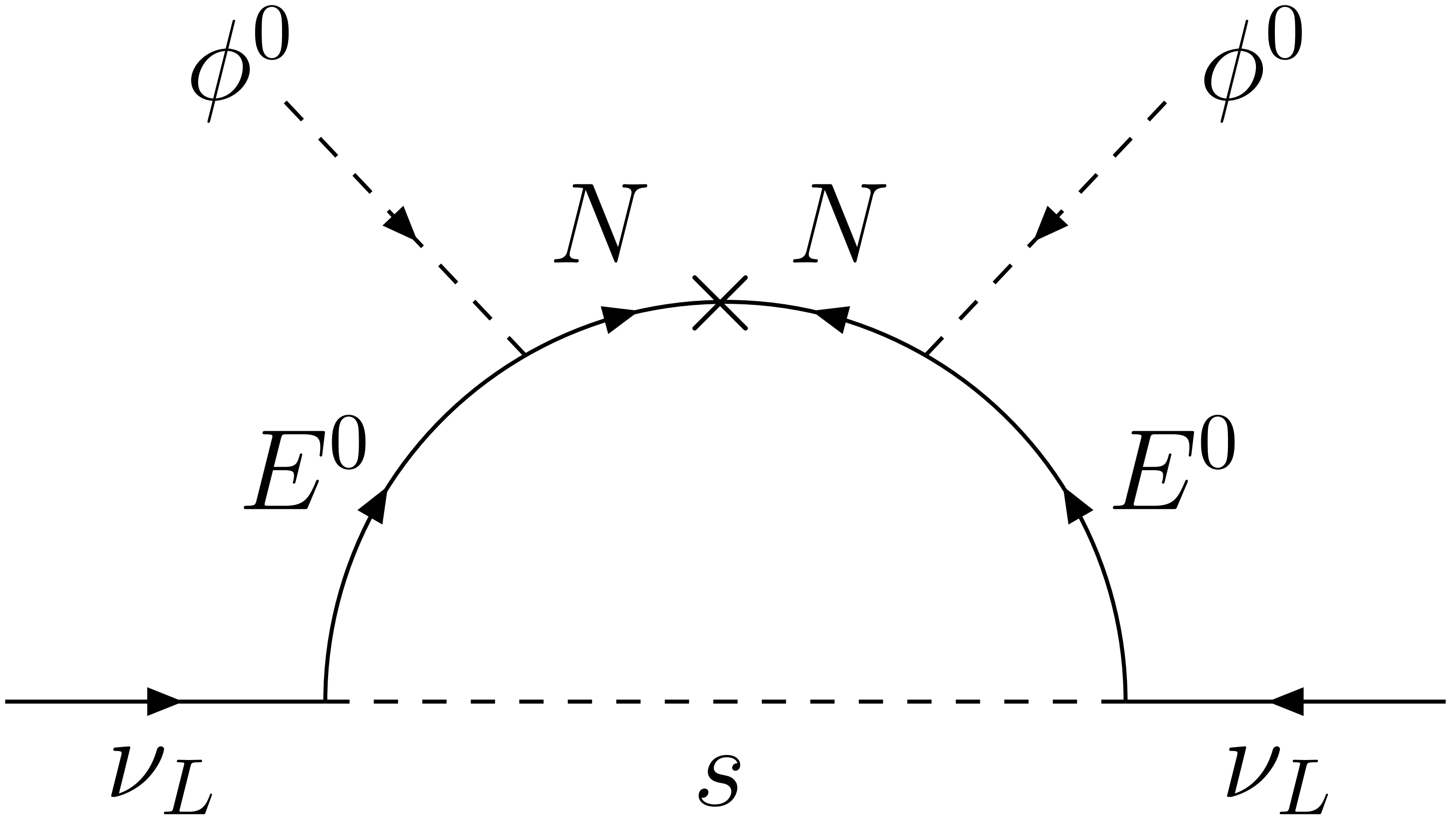
Obviously $\mathcal{O}$ would come from diagonalizing a real mass matrix. So if $\mathcal{M}_\nu$ is somehow purely real in the A_4 basis, then

$$\mathcal{M}_\nu^{(e,\mu,\tau)} = U_\omega^\dagger \begin{pmatrix} a & c & e \\ c & d & b \\ e & b & f \end{pmatrix} U_\omega^* = \begin{pmatrix} A & C & C^* \\ C & D^* & B \\ C^* & B & D \end{pmatrix},$$

where $A = (a + 2b + 2c + d + 2e + f)/3$,
$B = (a - b - c + d - e + f)/3$,
$C = (a - b - \omega c + \omega^2 d - \omega^2 e + \omega f)/3$,
$D = (a + 2b + 2\omega c + \omega^2 d + 2\omega^2 e + \omega f)/3$.
Cobimaximal mixing is thus automatically obtained.

The Majorana neutrino mass matrix is in general complex, so how does one guarantee it to be real? The answer was already there in a radiative inverse seesaw model of neutrino mass [Fraser/Ma/Popov(2014), Ma/Natale/Popov(2015)], where the origin of the neutrino mass matrix is that of a real scalar mass-squared matrix.

Actually the neutrino mass eigenvalues may pick up phases from the parameters involved in the loop calculation, but to obtain $|U_{\mu i}| = |U_{\tau i}|$, all that is required is for $\mathcal{M}_\nu$ to be diagonalized by $\mathcal{O}$.


ϕ^0
ϕ^0
N
N
E^0
E^0
ν_L
s
ν_L

Personal Remarks

Neutrino theory attempts to answer several fundamental questions, foremost is the scale of new physics responsible for neutrino mass and mixing. A possible hint is that they may also be connected to dark matter at the mass scale of 1 TeV. In the scotogenic framework, the A_4 transformation U_ω may be used to obtain a desirable cobimaximal form of $U_{l\nu}$, i.e. $\theta_{23} = \pi/4$ and $\delta_{CP} = \pm\pi/2$, automatically if the origin of the neutrino mass matrix is a set of real scalars $s_i \sim \underline{3}$ under A_4. Only one Higgs doublet with $\langle\phi^0\rangle$ is required.