Dark Revelations of $[SU(3)]^3$ and $[SU(3)]^4$

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Contents

Inroduction

Instead of $SU(5) \sim E_4$, or $SO(10) \sim E_5$, or E_6 unification, $[SU(N)]^k$ unification is an attractive alternative, with fermions transforming as

$$(N, N^*, 1, ...) + (1, N, N^*, ...) + ... + (N^*, 1, ..., N)$$

in a closed chain. Some random facts:

- (1) $SU(3)_C \times SU(3)_L \times SU(3)_R$ is the maximum subgroup of E_6:
  $\underline{27} = (3, 3^*, 1) + (1, 3, 3^*) + (3^*, 1, 3)$

- (2) $[SU(3)]^4$ may contain leptonic color: $SU(3)_q \times SU(3)_L \times SU(3)_l \times SU(3)_R$

- (3) $[SU(3)]^6$ may contain axial QCD and quark-lepton nonuniversality: $SU(3)_{CL} \times SU(3)_{qL} \times SU(3)_{lL} \times SU(3)_{lR} \times SU(3)_{qR} \times SU(3)_{CR}$

- (4) Supersymmetric $[SU(N)]^k$ is a finite field theory for 3 families independent of N and k:

$$b_i = -\frac{11}{3}N + \frac{2}{3}N + N_f\left(\frac{2}{3} + \frac{1}{3}\right)\frac{1}{2}(2N)$$

$[SU(3)]^3$ with Dark Symmetry

$$q \sim (3, 3^*, 1) \sim \begin{pmatrix} d & u & h \\ d & u & h \\ d & u & h \end{pmatrix},$$

$$\lambda \sim (1, 3, 3^*) \sim \begin{pmatrix} N & E^c & \nu \\ E & N^c & e \\ \nu^c & e^c & S \end{pmatrix},$$

$$q^c \sim (3^*, 1, 3) \sim \begin{pmatrix} d^c & d^c & d^c \\ u^c & u^c & u^c \\ h^c & h^c & h^c \end{pmatrix}.$$

The electric charge is $Q = I_{3L} - Y_L/2 + I_{3R} - Y_R/2$.
Consider now $D_A = 3(Y_L - Y_R)$.
The $[Q, D_A]$ assignments are

$$Q_q = \begin{pmatrix} -1/3 & 2/3 & -1/3 \\ -1/3 & 2/3 & -1/3 \\ -1/3 & 2/3 & -1/3 \end{pmatrix}, \quad D_q = \begin{pmatrix} -1 & -1 & 2 \\ -1 & -1 & 2 \\ -1 & -1 & 2 \end{pmatrix},$$

$$Q_\lambda = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \quad D_\lambda = \begin{pmatrix} 2 & 2 & -1 \\ 2 & 2 & -1 \\ -1 & -1 & -4 \end{pmatrix},$$

$$Q_{q^c} = \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ -2/3 & -2/3 & -2/3 \\ 1/3 & 1/3 & 1/3 \end{pmatrix}, \quad D_{q^c} = \begin{pmatrix} -1 & -1 & -1 \\ -1 & -1 & -1 \\ 2 & 2 & 2 \end{pmatrix}.$$

Define $R_A = (-1)^{D_A+2j}$, then $u, u^c, d, d^c, \nu, \nu^c, e, e^c$ are even and $h, h^c, N, N^c, E, E^c, S$ are odd. Hence R_A is a dark Z_2 parity, provided that D_A is only broken by 2 units.

The breaking of $SU(3)_L \times SU(3)_R$ is accomplished by a scalar bitriplet ϕ, transforming as λ, which also gives masses to all the fermions.

Now $\langle\phi_{33}\rangle$ breaks it to $SU(2)_L \times SU(2)_R \times U(1)_{Y_L+Y_R}$. Together with $\langle\phi_{11}\rangle, \langle\phi_{22}\rangle$, $SU(2)_L \times SU(2)_R$ is broken to $U(1)_{I_{3L}+I_{3R}}$.
At the same time D_A is broken to R_A, as desired.

However, the $SU(2)_R$ scale is not separated from the $SU(2)_L$ scale, and an unbroken symmetry other than $U(1)_Q$ remains.

To cure these problems while preserving R_A, an extra bitriplet η which has an extra overall odd R_A is added. It does not couple to the fermions, and allows us to define $\eta_{31}, \eta_{32}, \eta_{13}, \eta_{23}$ to be even and the other elements to be odd, so that the R_A parities of the gauge bosons are retained.

Now $\langle \eta_{31} \rangle$ breaks $SU(2)_R$ without breaking $SU(2)_L$, and together with $\langle \eta_{13} \rangle$, only $U(1)_Q$ remains unbroken.

To summarize

$$\begin{pmatrix} \phi^0_{11} & \phi^+_{12} & \phi^0_{13} \\ \phi^-_{21} & \phi^0_{22} & \phi^-_{23} \\ \phi^0_{31} & \phi^+_{32} & \phi^0_{33} \end{pmatrix}, \quad \begin{pmatrix} \eta^0_{11} & \eta^+_{12} & \eta^0_{13} \\ \eta^-_{21} & \eta^0_{22} & \eta^-_{23} \\ \eta^0_{31} & \eta^+_{32} & \eta^0_{33} \end{pmatrix}$$

The vacuum expectation values come from the scalars with even R_A: ϕ^0_{33}, η^0_{31}, ϕ^0_{22}, ϕ^0_{11}, η^0_{13}.

The singlet scalar η^0_{33} of odd R_A becomes a good dark matter candidate.

$[SU(3)]^4$ with Dark Symmetry

Leptonic color $SU(3)_l$ is the analog of quark color $SU(3)_q$. Whereas the latter is unbroken, the former is broken to $SU(2)_l$ which is itself unbroken. Of the fermion triplet under $SU(3)_l$, one component is free, i.e. the observed lepton, the other two (hemions) have half-integral charges and are confined by the massless $SU(2)_l$ gauge bosons (stickons), in analogy with the quarks being confined by the massless $SU(3)_q$ gluons.

$$l \sim \begin{pmatrix} x_1 & x_2 & \nu \\ y_1 & y_2 & e \\ z_1 & z_2 & n \end{pmatrix}, \quad l^c \sim \begin{pmatrix} x_1^c & y_1^c & z_1^c \\ x_2^c & y_2^c & z_2^c \\ \nu^c & e^c & n^c \end{pmatrix}.$$

The electric charge is now

$$Q = I_{3L} - \frac{Y_L}{2} + I_{3R} - \frac{Y_R}{2} - \frac{Y_l}{2}.$$

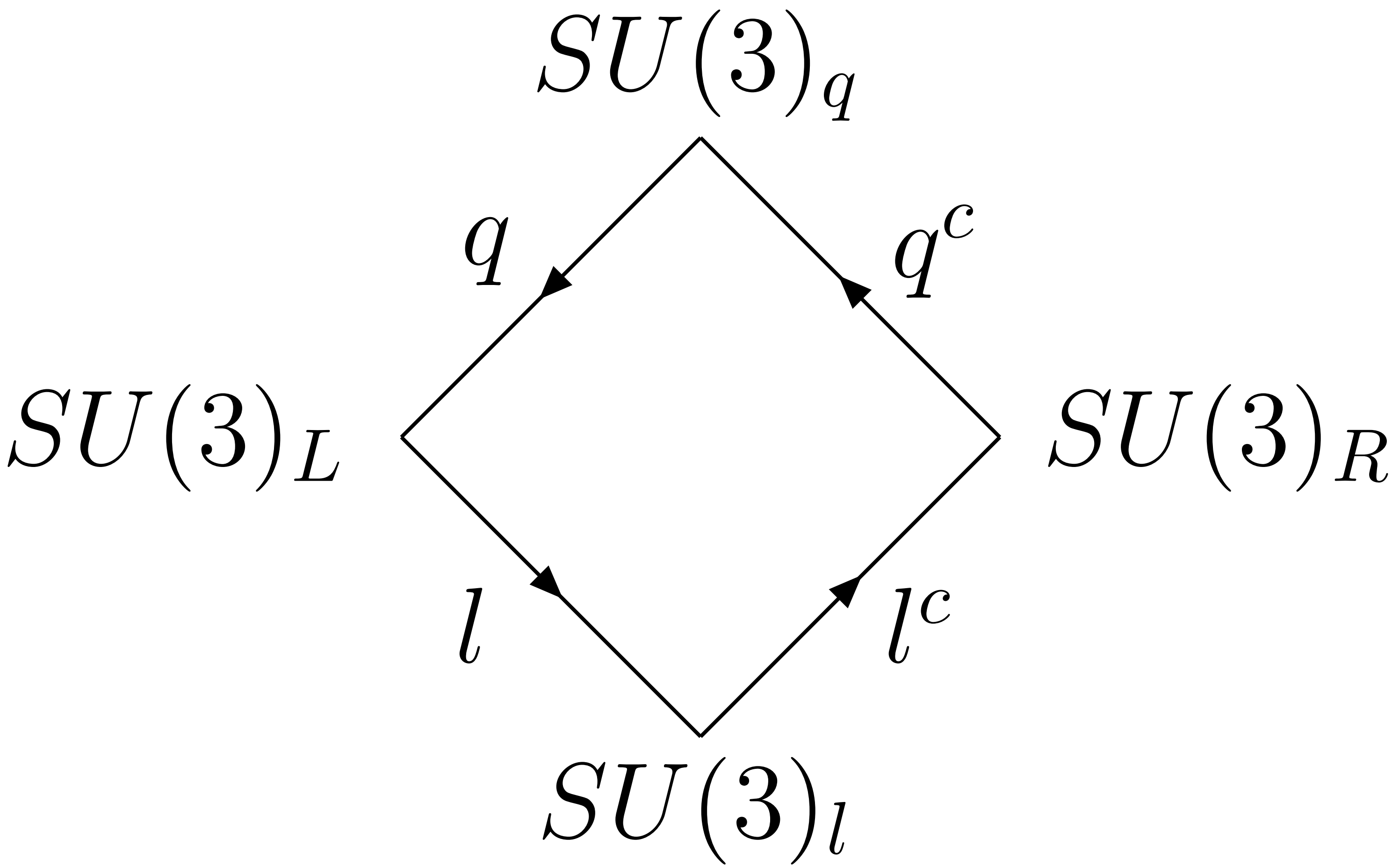
The dark charge remains defined by $D_A = 3(Y_L - Y_R)$. The $[Q, D_A]$ assignments for l, l^c are

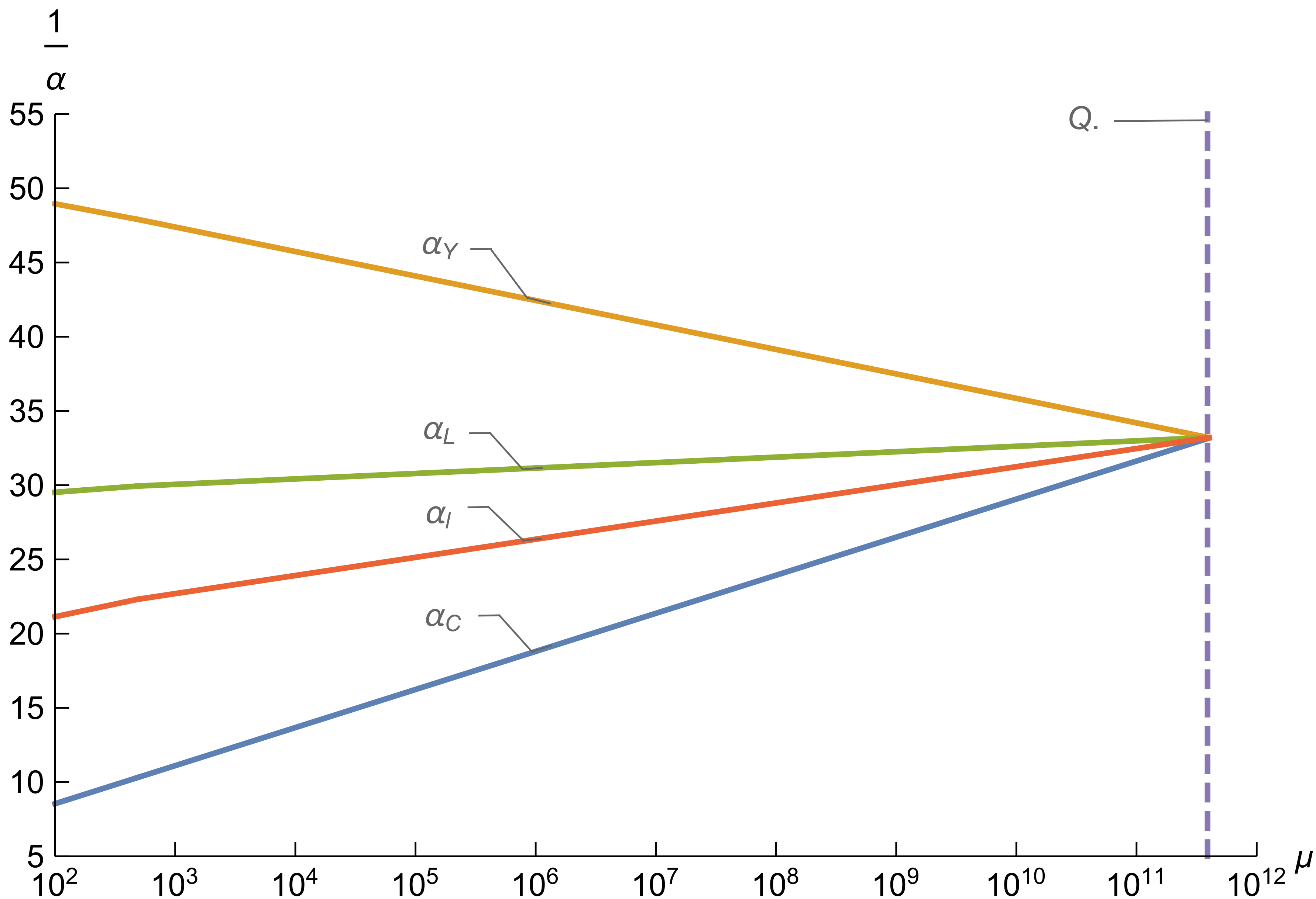
$$Q_l = \begin{pmatrix} 1/2 & 1/2 & 0 \\ -1/2 & -1/2 & -1 \\ 1/2 & 1/2 & 0 \end{pmatrix}, \quad D_l = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ -2 & -2 & -2 \end{pmatrix},$$

$$Q_{l^c} = \begin{pmatrix} -1/2 & 1/2 & -1/2 \\ -1/2 & 1/2 & -1/2 \\ 0 & 1 & 0 \end{pmatrix}, \quad D_{l^c} = \begin{pmatrix} 1 & 1 & -2 \\ 1 & 1 & -2 \\ 1 & 1 & -2 \end{pmatrix}.$$

The breaking of $SU(3)_L \times SU(3)_R$ proceeds as before by the ϕ bitriplet, which is responsible for fermion masses.

In addition, two scalar bitriplets are needed:
$\phi^L \sim l$ to break $SU(3)_l \times SU(3)_L$ to $SU(2)_l \times SU(2)_L \times U(1)_{Y_l+Y_L}$, and $\phi^R \sim l^c$ with an odd Z_2 to break $SU(2)_R$ without breaking $SU(2)_L$ as in the $[SU(3)]^3$ case, but η is not needed. Note that $\sin^2\theta_W = 1/3$ at the unification scale for $[SU(3)]^4$ instead of $3/8$ for $[SU(3)]^3$ or $[SU(3)]^6$. For the nonsupersymmetric Babu/Ma/Willenbrock model based on $[SU(3)]^4$, the unification scale is of order 10^{11} GeV.


$SU(3)_q$
q
q^c
$SU(3)_L$
$SU(3)_R$
l
l^c
$SU(3)_l$



$\frac{1}{\alpha}$
55
50
45
40
35
30
25
20
15
10
5
α_Y
α_L
α_I
α_C
Q.
10^2
10^3
10^4
10^5
10^6
10^7
10^8
10^9
10^{10}
10^{11}
10^{12}
μ

Three $SU(2)_R$ Variations

In analogy to the old flavor $SU(3)$ decompositions of (u, d, s) to T, V, U spins, there are three variations of the above models according to how $SU(2)_R$ is chosen. In the conventional approach (A), (u, d) is an $SU(2)_R$ doublet. Alternatively (B), (u, h) may be chosen. Thirdly (C), (h, d) is also possible. In $[SU(3)]^3$, (B) corresponds to

$$q^c \sim \begin{pmatrix} h^c & h^c & h^c \\ u^c & u^c & u^c \\ d^c & d^c & d^c \end{pmatrix}, \quad \lambda \sim \begin{pmatrix} \nu & E^c & N \\ e & N^c & E \\ S & e^c & \nu^c \end{pmatrix}.$$

Whereas Q is the same in (B) as in (A), the dark charge is instead $D_B = 3(Y_L + I_{3R} + Y_R/2)$. Now ϕ has nonzero vacuum expectation values for the $(13), (22), (31)$ entries, and η for $(11), (33)$.

The variation (C) corresponds to

$$q^c \sim \begin{pmatrix} d^c & d^c & d^c \\ h^c & h^c & h^c \\ u^c & u^c & u^c \end{pmatrix}, \quad \lambda \sim \begin{pmatrix} N & \nu & E^c \\ E & e & N^c \\ \nu & S & e^c \end{pmatrix}.$$

Here, $Q = I_{3L} - Y_L/2 + Y_R$ and $D_C = 3(Y_L - I_{3R} + Y_R/2)$. However, neither ϕ nor η could preserve $SU(2)_R$ as a low-energy subgroup.

In $[SU(3)]^4$, the leptonic sector mimics the quark sector exactly, except for the charges. Whereas q and l remain the same for all variations, q^c varies as in $[SU(3)]^3$ and l^c transforms as

$$(B) \sim \begin{pmatrix} z_1^c & y_1^c & x_1^c \\ z_2^c & y_2^c & x_2^c \\ n^c & e^c & \nu^c \end{pmatrix}, \quad (C) \sim \begin{pmatrix} x_1^c & z_1^c & y_1^c \\ x_2^c & z_2^c & y_2^c \\ \nu^c & n^c & e^c \end{pmatrix}.$$

In all variations, compared to $[SU(3)]^3$, Q changes by $-Y_l/2$ and $D_{A,B,C}$ are the same. Again (C) cannot keep $SU(2)_R$ at low energy. As for (A), the dark particles are all very heavy $SU(2)_R$ singlets in $[SU(3)]^3$ or $[SU(3)]^4$.

Alternative $[SU(3)]^4$ Model of Dark Matter

Under $SU(3)_q \times SU(3)_L \times SU(3)_l \times SU(3)_R$ with $D = \sqrt{3}(2Y_L + \sqrt{3}I_{3R} + Y_R)$, where Y is normalized by $\sum Y^2 = 1/2$, the fermions are

$$q \sim (3, 3^*, 1, 1) \sim \begin{pmatrix} d & u & h \\ d & u & h \\ d & u & h \end{pmatrix}, \quad D_q \sim \begin{pmatrix} -1 & -1 & 2 \\ -1 & -1 & 2 \\ -1 & -1 & 2 \end{pmatrix},$$

$$l \sim (1, 3, 3^*, 1) \sim \begin{pmatrix} x_1 & x_2 & \nu \\ y_1 & y_2 & e \\ z_1 & z_2 & n \end{pmatrix}, \quad D_l \sim \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ -2 & -2 & -2 \end{pmatrix},$$

$$l^c \sim (1,1,3,3^*) \sim \begin{pmatrix} z_1^c & y_1^c & x_1^c \\ z_2^c & y_2^c & x_2^c \\ n^c & e^c & \nu^c \end{pmatrix}, \quad D_{l^c} \sim \begin{pmatrix} -2 & 1 & 1 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{pmatrix},$$

$$q^c \sim (3^*,1,1,3) \sim \begin{pmatrix} h^c & h^c & h^c \\ u^c & u^c & u^c \\ d^c & d^c & d^c \end{pmatrix}, \quad D_{q^c} \sim \begin{pmatrix} 2 & 2 & 2 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \end{pmatrix}.$$

Using $R_D = (-1)^{D+2j}$, we see that $u, u^c, d, d^c, \nu, \nu^c, e, e^c, x, x^c, y, y^c$ are even, and h, h^c, z, z^c, n, n^c are odd. Further, the gauge bosons which take h to u, d and h^c to u^c, d^c are odd and the others even. Hence R_D would remain unbroken if the scalars breaking the gauge symmetry are even.

The scalar bitriplets responsible for the masses of the above fermions come from three chains, each of the form $(3,1,3^*,1)+(1,3,1,3^*)+(3^*,1,3,1)+(1,3^*,1,3)$. Specifically,

$$\phi^{(1,3,5)} \sim (1,3,1,3^*) \sim \begin{pmatrix} \eta^0 & \phi_2^+ & \phi_1^0 \\ \eta^- & \phi_2^0 & \phi_1^- \\ \chi^0 & \chi^+ & \lambda^0 \end{pmatrix}, \quad D_\phi \sim \begin{pmatrix} -1 & 2 & 2 \\ -1 & 2 & 2 \\ -4 & -1 & -1 \end{pmatrix},$$

$$\bar\phi^{(2,4,6)} \sim (1,3^*,1,3) \sim \begin{pmatrix} \bar\eta^0 & \eta^+ & \bar\chi^0 \\ \phi_2^- & \bar\phi_2^0 & \chi^- \\ \bar\phi_1^0 & \phi_1^+ & \bar\lambda^0 \end{pmatrix}, \quad D_{\bar\phi} \sim \begin{pmatrix} 1 & 1 & 4 \\ -2 & -2 & 1 \\ -2 & -2 & 1 \end{pmatrix}.$$

$\chi^0 \to hh^c$, $\phi_1^0 \to dd^c$, $\phi_2^0 \to uu^c$,

$\bar\chi^0 \to nn^c, zz^c$, $\bar\phi_1^0 \to \nu\nu^c, xx^c$, $\bar\phi_2^0 \to ee^c, yy^c$.

To break $[SU(3)]^4$ at M_U to $SU(3)_q \times SU(2)_l \times SU(2)_L \times SU(2)_R \times U(1)_X \times R_D$, where $X = (Y_L + Y_R + Y_l)/\sqrt{3}$ with $Q = I_{3L} + I_{3R} - X$, two scalar bitriplets are used. Both come from chains transforming as the fermions. The first is $\phi^{L+} \sim (1, 3, 3^*, 1) \sim l$ with the same R_D assignments. Hence $\langle \phi^{L+}_{33} \rangle$ breaks $SU(3)_L \times SU(3)_l$ to $SU(2)_L \times SU(2)_l \times U(1)_{(Y_L+Y_l)/\sqrt{2}} \times R_D$.

Note that the allowed antisymmetric trilinear term $ll\phi^{L+}$ implies the existence of a superheavy mass term for $x_1 y_2 - x_2 y_1$.

The second is $\phi^{R-} \sim (1,1,3,3^*) \sim l^c$ but with an overall odd parity, which may be absorbed into R_D so that ϕ^{R-}_{33} is even and $\langle \phi^{R-}_{33} \rangle$ breaks $SU(3)_l \times SU(3)_R$ to $SU(2)_l \times SU(2)_R \times U(1)_{(Y_R+Y_l)/\sqrt{2}} \times R_D$. Together they achieve the desirable symmetry breaking at M_U.

Note that the would-be antisymmetric $l^c l^c \phi^{R-}$ term is forbidden because of the extra odd parity. It means that the mass term $z^c_1 y^c_2 - z^c_2 y^c_1$ is not generated, but rather the mass term $x^c_1 y^c_2 - x^c_2 y^c_1$ appears at the $SU(2)_R$ symmetry breaking scale M_R through ϕ^{R+}_{31} as part of the chain containing ϕ^{L+}.

Since there are three fermion chains, and five scalar chains, the b coefficients for the renormalization-group running of each $SU(3)$ gauge coupling are all given by

$$b = -11 + \frac{2}{3}\left(\frac{1}{2}\right)(2)(3)(3) + \frac{1}{3}\left(\frac{1}{2}\right)(2)(3)(5) = 0.$$

This shows that a possible finite theory exists above the M_U scale. At M_R, the $SU(2)_R \times U(1)_X$ gauge symmetry is broken to $U(1)_Y$, where $Y = I_{3R} - X$, by an $SU(2)_R$ doublet whose neutral component is a linear combination of χ^0 from $\phi^{(1)}$, the conjugate of $\bar{\chi}^0$ from $\bar{\phi}^{(2)}$, and ϕ_{31}^{R+}.

The renormalization-group evolution of the gauge couplings is dictated at leading order by

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(\mu')} + \frac{b_i}{2\pi} \ln\left(\frac{\mu'}{\mu}\right),$$

where b_i are the one-loop beta-function coefficients.

From M_U to M_R, we assume all fermions are light except the three families of (x, y) hemions. As for scalars, under $SU(2)_L \times SU(2)_R \times U(1)_X$, we assume 1 copy of $(1, 2, -1/2)$, 6 copies of $(2, 2, 0)$, 4 copies of $(2, 1, 1/2)$, and 3 copies of $(2, 1, -1/2)$.

$$b_q = -11 + \frac{2}{3}\left(\frac{1}{2}\right)(6)(3) = -5,$$
$$b_l = -\frac{22}{3} + \frac{2}{3}\left(\frac{1}{2}\right)(4)(3) = -\frac{10}{3},$$
$$b_L = -\frac{22}{3} + \frac{2}{3}\left(\frac{1}{2}\right)(3+1)(3) + \frac{1}{3}\left(\frac{1}{2}\right)[7+6(2)] = -\frac{1}{6},$$
$$b_R = -\frac{22}{3} + \frac{2}{3}\left(\frac{1}{2}\right)(3+2+1)(3) + \frac{1}{3}\left(\frac{1}{2}\right)[1+6(2)] = \frac{5}{6},$$
$$b_X = \frac{2}{3}\left[\frac{1}{6}(3) + \frac{1}{6}(3) + \frac{1}{4}(4) + \frac{1}{4}(4)\right](3) + \frac{1}{3}\left(\frac{1}{4}\right)[2+7(2)] = \frac{22}{3}.$$

From M_R to M_Z, the SM particle content is assumed together with 1 copy of (x^c, y^c) hemions and two $SU(2)_L$

Higgs scalar doublets. The massless $SU(2)_l$ stickons are of course included but they affect only α_l. The four b coefficients are then

$$b_q = -11 + \frac{2}{3}\left(\frac{1}{2}\right)(4)(3) = -7,$$

$$b_l = -\frac{22}{3} + \frac{2}{3}\left(\frac{1}{2}\right)(2) = -\frac{20}{3},$$

$$b_L = -\frac{22}{3} + \frac{2}{3}\left(\frac{1}{2}\right)(3+1)(3) + \frac{1}{3}\left(\frac{1}{2}\right)(2) = -3,$$

$$b_Y = \frac{1}{2}\left[\frac{2}{3}\left\{\frac{10}{3}(3) + \frac{1}{4}(4)\right\} + \frac{1}{3}\left(\frac{1}{4}\right)(4)\right] = \frac{23}{6},$$

where a factor of $1/2$ has been inserted to normalize b_Y. At M_R,

$$\frac{2}{\alpha_Y(M_R)} = \frac{1}{\alpha_R(M_R)} + \frac{1}{\alpha_X(M_R)}.$$

Combining the above,

$$\frac{1}{\alpha_q(M_Z)} = \frac{1}{\alpha_U} - \frac{7}{2\pi}\ln\frac{M_R}{M_Z} - \frac{5}{2\pi}\ln\frac{M_U}{M_R},$$

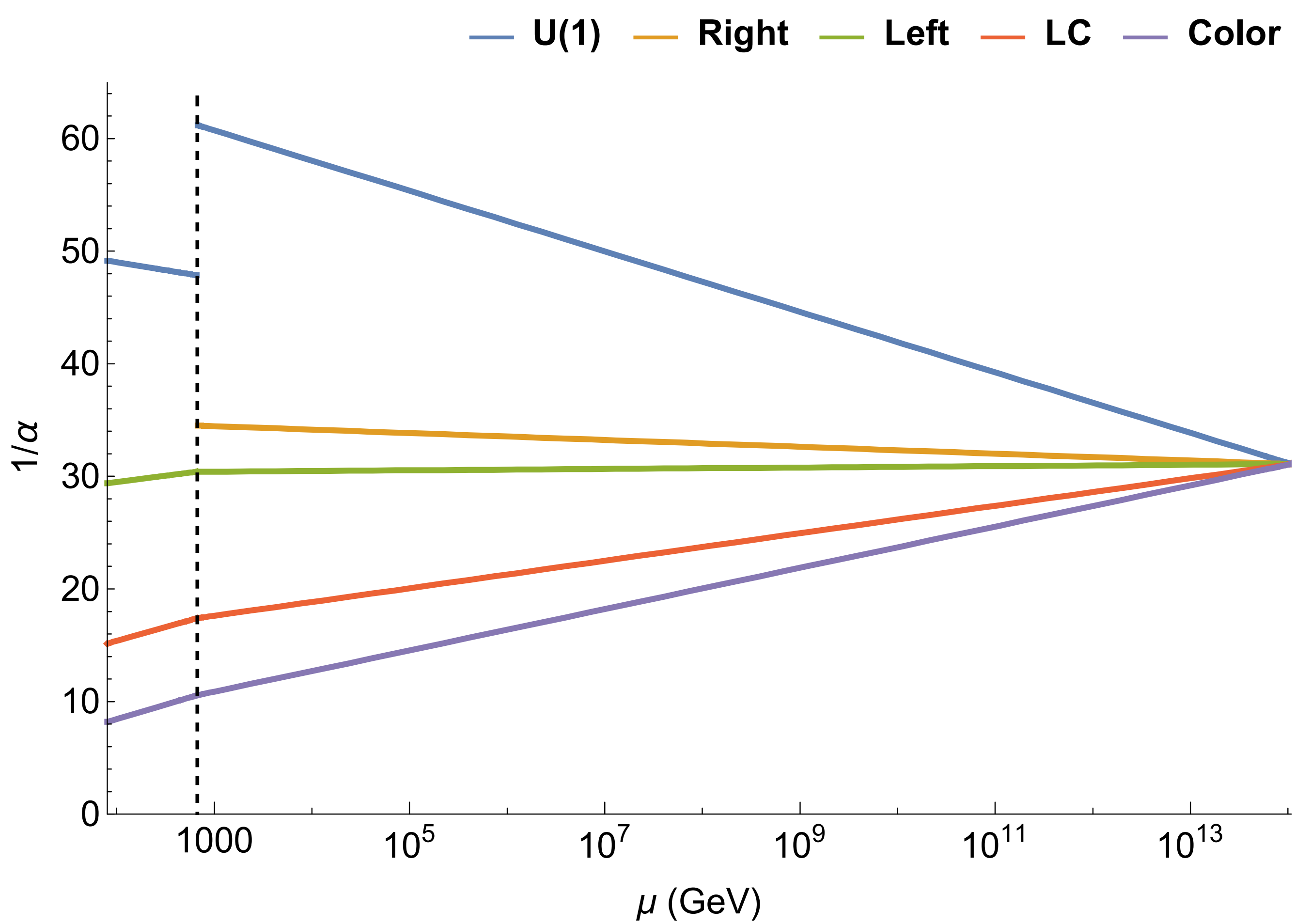
$$\frac{1}{\alpha_L(M_Z)} = \frac{1}{\alpha_U} - \frac{3}{2\pi}\ln\frac{M_R}{M_Z} - \frac{1}{6(2\pi)}\ln\frac{M_U}{M_R},$$

$$\frac{1}{\alpha_Y(M_Z)} = \frac{1}{\alpha_U} + \frac{23}{6(2\pi)}\ln\frac{M_R}{M_Z} + \frac{49}{12(2\pi)}\ln\frac{M_U}{M_R}.$$

Using the experimental inputs $\alpha_q(M_Z) = 0.1185$,
$\alpha_L(M_Z) = (\sqrt{2}/\pi) G_F M_W^2 = 0.0339$, and
$\alpha_Y(M_Z) = 2\alpha_L(M_Z) \tan^2\theta_W = 0.0204$,
where a factor of 2 has been used to normalize α_Y, we find $M_R \simeq 600$ GeV and $M_U \simeq 10^{14}$ GeV.
As a result, $\alpha_U = 0.0322$. Using

$$\frac{1}{\alpha_l(M_Z)} - \frac{1}{\alpha_q(M_Z)} = \frac{1}{3(2\pi)} \ln\frac{M_R}{M_Z} + \frac{5}{3(2\pi)} \ln\frac{M_U}{M_R},$$

the leptonic color $\alpha_l(M_Z)$ is determined to be 0.0650, implying a QHD confining scale of about 0.4 MeV.
Similarly, $\alpha_R(M_R) = 0.0290$ is also obtained.


U(1)
Right
Left
LC
Color
60
50
40
30
20
10
0
1/α
1000
10^5
10^7
10^9
10^11
10^13
μ (GeV)

The particles of this model at or below a few TeV are

particles	$S=(Y_R-2Y_L)/\sqrt{3}$	$I_{3R}+S$
$(u,d)_L, d_R$ $(u,h)_R, h_L$	1/3 $(-1/6,-1/6),-2/3$	1/3 $(1/3,-2/3),-2/3$
$(\nu,l)_L, \nu_R, x_R$ $(n,l)_R, n_L$ $(z,y)_R, z_L$	$-1/3$ $(1/6,1/6),2/3$ $(1/6,1/6),2/3$	$-1/3$ $(2/3,-1/3),2/3$ $(2/3,-1/3),2/3$
(ϕ_1^0,ϕ_1^-) (χ^+,χ^0) (η,Φ_2) λ^0	0 1/2 $-1/2$ 1	0 (1,0) $(-1,0)$ 1

The scalar bidoublet contains two $SU(2)_L$ doublets: $\eta = (\eta^0, \eta^-)$ with $I_{3R} + S = -1$ and $\Phi_2 = (\phi_2^+, \phi_2^0)$ with $I_{3R} + S = 0$. Now

$$I_{3R} + S = B - \frac{1}{3}L,$$

for the SM quarks and leptons. Recall that in $SU(5)$, neither B nor L is part of $SU(5)$, but both are good low-energy symmetries. Here, B and L are also not part of $[SU(3)]^4$ by themselves, but the linear combination $B - (L/3)$ is, with $R_D = (-1)^{3B-L+2j}$, even though its corresponding gauge symmetry is broken.

At temperatures above 100 GeV or so, the hemions (x_R, y_R) are active and the stickons ζ [$SU(2)_l$ gauge bosons] are in thermal equilibrium with the SM particles. Below 100 GeV, the stickon interacts with photons through $\zeta\zeta \to \gamma\gamma$ scattering with a cross section

$$\sigma \sim \frac{\alpha^2 \alpha_l^2 T^6}{64 m^8},$$

where m is the hemion mass. The decoupling temperature of ζ is then obtained by matching the Hubble expansion rate $H = \sqrt{(8\pi/3) G_N (\pi^2/30) g_* T^4}$

to $[6\zeta(3)/\pi^2]T^3\langle\sigma v\rangle$. Hence

$$T^{14} \sim \frac{2^{12}}{3^4}\left(\frac{\pi^7}{5[\zeta(3)]^2}\right)\frac{G_N g_* m^{16}}{\alpha^4\alpha_l^4}.$$

For $m = 100$ GeV and $g_* = 92.25$ which includes all particles with masses up to a few GeV, $T \sim 9$ GeV. Hence the contribution of stickons to N_{eff} of neutrinos at the time of BBN is

$$\Delta N_\nu = \frac{8}{7}(3)\left(\frac{10.75}{92.25}\right)^{4/3} = 0.195,$$

compared to the value 0.50 ± 0.23 from a 2015 analysis.

As the Universe further cools below a few MeV, leptonic color goes through a phase transition and stickballs ω are formed. They may mix with a scalar bound state of hemions and decay to two photons, with the rate

$$\Gamma(\omega \to \gamma\gamma) = \frac{\alpha^2 f_\omega^2 m_\omega^2}{256\pi^3 m^4}.$$

Using $m_\omega = 1$ MeV with $m = 100$ GeV, its lifetime is 10^7 s for $f_\omega = 1$. This means that it disappears long before the time of photon decoupling. Hence N_{eff} remains the same as in the SM for the CMB, i.e. 3.046, which is consistent with the PLANCK result of 3.15 ± 0.23.

Unlike quarks, all hemions are heavy. The lightest vector bound state Ω is likely to be at least 200 GeV. Its production cross section at the LHC is small and the background is large. However, at a future e^-e^+ collider, it may be discovered at resonance, in analogy to J/ψ and Υ for quarks. The Bohr radius for a hemionium is $a_0 = [(3/8)\bar{\alpha}_l m]^{-1}$, where $\bar{\alpha}_l = \alpha_l(a_o^{-1})$. For $\alpha = 0.065$ and $m = 100$ GeV, we obtain $\bar{\alpha}_l = 0.087$ and $a_0^{-1} = 3.26$ GeV. The ground state has binding energy

$$E_b = \frac{1}{4}\left(\frac{3}{4}\right)^2 \bar{\alpha}_l^2 m = 106 \text{ MeV},$$

and its wavefunction at the origin is $|\psi(0)|^2 = (\pi a_0^3)^{-1} = 11.03$ GeV^3. Its integrated cross section over the energy range $\sqrt{s}$ around m_Ω is

$$\int d\sqrt{s}\sigma(e^- e^+ \to \Omega \to X) = \frac{6\pi^2}{m_\Omega^2}\frac{\Gamma_{ee}\Gamma_X}{\Gamma_{tot}}.$$

Now Ω will decay through the photon and the Z boson to $W^- W^+, q\bar{q}, l^- l^+, \nu\bar{\nu}$. Using $\langle 0|\bar{x}\gamma^\mu x|\Omega\rangle = \epsilon_\Omega^\mu \sqrt{8m_\Omega}|\psi(0)|$, then

$$\Gamma(\Omega \to e^- e^+) = \frac{2m_\Omega^2}{3\pi}(|C_V|^2 + |C_A|^2)|\psi(0)|^2.$$

Ω may also decay to 3 stickons and 2 stickons + 1 photon:

$$\Gamma(\Omega \to \zeta\zeta\zeta) = \frac{16}{27}(\pi^2 - 9)\frac{\alpha_l^3}{m_\Omega^2}|\phi(0)|^2,$$

$$\Gamma(\Omega \to \gamma\zeta\zeta) = \frac{8}{9}(\pi^2 - 9)\frac{\alpha\alpha_l^2}{m_\Omega^2}|\psi(0)|^2.$$

The integrated cross section for $X = \mu^-\mu^+$ is then 1.2×10^{-32} cm^2-keV.

For comparison, this number is 7.9×10^{-30} cm^2-keV for the $\Upsilon(1S)$.

Channel	Width
$\sum \nu\bar{\nu}$	36 eV
$e^-e^+, \mu^-\mu^+, \tau^-\tau^+$	0.4 keV
$u\bar{u}, c\bar{c}$	0.3 keV
$d\bar{d}, s\bar{s}, b\bar{b}$	0.1 keV
W^-W^+	10 eV
ZZ	< 0.1 eV
$\zeta\zeta\zeta$	39 eV
$\gamma\zeta\zeta$	7 eV
sum	0.9 keV

There are important differences between QCD and QHD (quantum hemiodynamics). In the former, because of the existence of light u and d quarks, it is easy to pop up $u\bar{u}$ and $d\bar{d}$ pairs from the QCD vacuum. Hence the production of open charm is described well by the process $e^-e^+ \to c\bar{c}$. In the latter, there are no light hemions. Instead it is easy to pop up the light stickballs from the QHD vacuum, just above the threshold of making the Ω resonance. This cross section σ is presumably also well described by $e^-e^+ \to x\bar{x}$. For $m = 100$ GeV and $\sqrt{s} = 250$ GeV, $\sigma = 0.79$ pb.

In QCD, there are $q\bar{q}$ bound states which are bosons, and qqq bound states which are fermions. In QHD, there are only bound-state bosons, because the confining symmetry is $SU(2)_l$. Also, unlike baryon (or quark) number in QCD, there is no such thing as hemion number in QHD, because y is effectively $\bar{x}$. This explains why there are no stable analog fermion in QHD such as the proton in QCD.

Hemions are not easily produced at the LHC. They are however very suited to be observed as hemionium resonances at a future high-energy e^-e^+ collider.

As the model stands, neutrinos are Dirac particles because of the conserved $B - (L/3)$ symmetry. To obtain Majorana neutrino masses, three singlet fermions N with even R_D may be added. The terms $N\nu^c(\phi_{33}^{R-})^*$ and $N\nu(\phi_{13}^{L-})^*$ are then allowed, and together with the $\nu\nu^c$ Dirac mass and the NN Majorana mass, the 3×3 mass matrix spanning (ν, ν^c, N) is of the form

$$\mathcal{M}_{\nu,\nu^c,N} = \begin{pmatrix} 0 & m_1 & m_2 \\ m_1 & 0 & M_1 \\ m_2 & M_1 & M_2 \end{pmatrix},$$

with a seesaw mass $m_\nu \simeq (m_1^2 M_2 - 2m_1 m_2 M_1)/M_1^2$.

The $SU(2)_R$ charged gauge bosons $W_R^{\pm}$ have odd R_D and belong to the dark sector. They do not mix with the SM $W_L^{\pm}$. In the neutral sector, in addition to the photon and the SM Z boson, there is $Z' = (g_R^2 + g_X^2)^{-1/2}(g_R W_{3R} - g_X Z_X)$, with

$$\mathcal{L}_{Z'} = (g_R^2 + g_X^2)^{-1/2} Z'_\mu (g_R^2 j_{3R}^\mu + g_X^2 j_X^\mu).$$

The $Z - Z'$ mixing is proportional to $(g_X^2 v_1^2 - g_R^2 v_2^2)/v_R^2$ and may be chosen to the negligible. The current LHC bound on $M_{Z'}$ is expected to be a few TeV.

Possible dark matter candidates are the fermion n, and the scalars η^0, λ^0. Now η^0 couples to Z and n couples to Z', so they interact with nuclei. The former is ruled out already by orders of magnitude in underground direct-search experiments. (The exception in scotogenic models comes from the splitting of the imaginary and real components of η^0.) The latter is severely constrained. This leaves the singlet λ^0. Its interaction with the SM Higgs boson alone allows it to be considered as a possible dark matter candidate. Here it also has the interactions: $\bar{d}_R h_L \lambda^0$, $\bar{n}_L \nu_R \lambda^0$, $\bar{z}_L x_R \lambda^0$, and $\lambda^0(\eta^0 \phi_2^0 - \eta^- \phi_2^+)$.

Conclusion

The existence of a dark symmetry is easily implemented by adding a new symmetry and new particles to the standard model. There are indeed numerous such proposals. Is there a guiding principle?

Supersymmetry with R parity is the first well-known example, except for its lack of supporting evidence.

Here an alternative guiding principle is explored, i.e. that such a dark symmetry may have a gauge origin buried inside a complete extended theoretical framework for the understanding of quarks and leptons.

In $[SU(3)]^3$ and $[SU(3)]^4$, there are dark gauge symmetries which have been overlooked. The nature of each in three variations has been identified. In every case, this $U(1)$ gauge symmetry is broken to dark Z_2 parity. The dark sector is an integral part of the complete theory and it includes fermions, scalars, and vector gauge bosons, in parallel to those of the SM. Only the variation (B) in either $[SU(3)]^3$ or $[SU(3)]^4$ has a low-energy $SU(2)_R$ for dark matter with a generalized conserved $B - (L/3)$. This study points to the unity of matter with dark matter, the origin of which is not *ad hoc*.