Jarlskog invariant in $SM \otimes S_3$

E. Barradas Guevara (BUAP), O. Félix-Beltrán (BUAP), J. Montano-Peraza (UNISON), E. Rodríguez-Jáuregui (UNISON)

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Abstract

CP violation in the Standard Model is related to the CP violating phase in the $\mathbf{V}_{\mathrm{CKM}}$ mixing matrix. The Jarlskog invariant J which measures CP violation can be expressed in terms of entries of CKM mixing matrix or in terms of the up and down quark mass matrices. In this work we show the Jarlskog invariant from an extended Standard Model with a permutational flavour symmetry S_3 (S_3SM). As one knows, CP-violation phase in the $\mathbf{V}_{\mathrm{CKM}}$ is enough to explain CP violating physics effects in the EW quark sector. On the other hand, an extended S_3 Higgs potential can be a source of spontaneous CP violation through the complex Higgs vacuum expectation values. Spontaneous CP violation effects contributes to the Higgs mass matrix as well as up and down quark mass matrices and the trilinear Higgs coupling parameters. Then, a non vanishing Jarlskog invariant J provides a necessary and sufficient condition for a spontaneous CPV coming from S_3SM.

Introduction

In the SM, only one $SU(2)_L$ doublet Higgs field is included,

$$\mathcal{L}_{SB} = \left(\mathcal{D}_\mu \Phi\right)^\dagger \left(\mathcal{D}^\mu \Phi\right) - V\left(\Phi\right) \tag{1}$$

$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda \left(\Phi^\dagger \Phi\right)^2 \quad \text{with} \quad \lambda > 0 \tag{2}$$

where

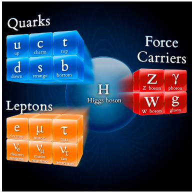
$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \tag{3}$$

which, upon acquiring a vacuum expectation value:

$$SU(2)_L \times U(1)_Y \to U(1)_{EM}$$

Introduction

Although the existence of the Higgs particle is a fundamental piece of the theory and the Higgs potential is very simple and sufficient to describe a realistic model of mass generation, this may not be the final form of the theory.



In the SM each family of fermions enters independently, in order to understand the replication of generations and to reduce the number of free parameters, usually more symmetry is introduced in the theory.

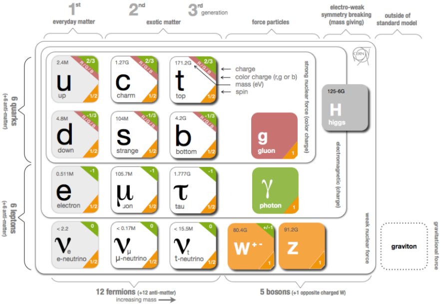
Introduction

It is noticeable that many interesting features of masses and mixing of the SM can be understood using a minimal discrete group, namely the permutational group S_3.[1]

[1] Derman:1978rx, Derman:1979nf, Pakvasa:1977in, Pakvasa:1978tx, Mondragon:1998yw, Mondragon:1998gy, Mondragon:1999jt, Harrison:2003aw, Kubo:2004ps, Caravaglios:2005gw,Araki:2005ec, Kubo:2005sr, Koide:2005ep, Grimus:2005mu, Teshima:2005bk, Kimura:2005sx, Koide:2006vs, Mohapatra:2006pu, Kaneko:2007ea, Beltran:2009zz,Morisi:2010rk.

The Standar Model

Prior to the introduction of the Higgs boson, the SM is chiral and invariant with respect to any permutation of the left and right quark and lepton fields. After the introduction of the Higgs boson in the theory, this field may be treated as an S_3 singlet H_S, but then, only one fermion in each family can acquire mass.



The Standar Model

In the SM, only one $SU(2)_L$ doublet Higgs field is included, which, upon acquiring a vacuum expectation value, breaks the $SU(2)_L \times U(1)_Y$ symmetry.
In the SM each family of fermions enters independently, in order to understand the replication of generations and to reduce the number of free parameters, usually more symmetry is introduced in the theory.
An extended Higgs sector opened up the window for CP violation scenarios coming from the Higgs sector, we look for the conditions under which CP violation arises from spontaneous gauge symmetry breaking $SU(2)_L \otimes U(1)_Y \to U(1)_{EM}$.
In this direction interesting work has been done with the addition of discrete symmetries to the SM.
It is noticeable that many interesting features of masses and mixing of the SM can be understood using a minimal discrete group, namely the permutational group $S(3)$.

The SM extended with $S(3)$

The Lagrangian $\mathcal{L}_\Phi$ of the Higgs sector is given by

$$\mathcal{L}_\Phi = [D_\mu H_S]^2 + [D_\mu H_1]^2 + [D_\mu H_2]^2 - V(H_1, H_2, H_S),$$

where D_μ is the usual covariant derivative. The scalar potential $V(H_1, H_2, H_S)$ is the most general Higgs potential invariant under $SU(3)_C \times SU(2)_L \times U(1)_Y \times S_3$.

The analysis of the stability properties of the potential V is of great relevance to study the phenomenological implications of this model. There are many different ways of writing the Higgs potential for this model, but for the purpose of this work the best basis is

$$H_1 = \begin{pmatrix} \phi_1 + i\phi_4 \\ \phi_7 + i\phi_{10} \end{pmatrix}, \; H_2 = \begin{pmatrix} \phi_2 + i\phi_5 \\ \phi_8 + i\phi_{11} \end{pmatrix},$$

$$H_S = \begin{pmatrix} \phi_3 + i\phi_6 \\ \phi_9 + i\phi_{12} \end{pmatrix}.$$

The Higgs Potential

$$\begin{aligned} V \;=\; & \mu_1^2 (x_1 + x_2) + \mu_0^2 x_3 + a x_3^2 + b (x_1 + x_2) x_3 + c (x_1 + x_2)^2 \\ & -4d x_7^2 + 2e \left[(x_1 - x_2) x_6 + 2 x_4 x_5\right] + f \left(x_5^2 + x_6^2 + x_8^2 + x_9^2\right) \\ & +g \left[(x_1 + x_2)^2 + 4 x_4^2\right] + 2h \left(x_5^2 + x_6^2 - x_8^2 - x_9^2\right) . \end{aligned} \tag{4}$$

where

- the $\mu_{0,1}^2$ parameters have dimensions of mass squared,
- the $a, \cdots, h$ parameters are dimensionless.

The invariants x_i, the potential V depends on the fields ϕ_i through x_i, considering our assignment as

$$\begin{aligned} x_1 &= H_1^\dagger H_1, & x_4 &= \mathcal{R}\left(H_1^\dagger H_2\right), & x_7 &= \mathcal{I}\left(H_1^\dagger H_2\right), \\ x_2 &= H_2^\dagger H_2, & x_5 &= \mathcal{R}\left(H_1^\dagger H_S\right), & x_8 &= \mathcal{I}\left(H_1^\dagger H_S\right), \\ x_3 &= H_S^\dagger H_S, & x_6 &= \mathcal{R}\left(H_2^\dagger H_S\right), & x_9 &= \mathcal{I}\left(H_2^\dagger H_S\right). \end{aligned} \tag{5}$$

Stationary points

We calculate now the stationary points of the potential (4) in order to determine their phenomenological feasibility [2]. We assume that H_S is the SM Higgs, and we do not break the electric charge nor CP (recall that $H_S \to \Phi_{SM}$) when H_S acquires a non zero vev. If there is CB or CP breaking in the model we are assuming that it is due to the S_3 Higgs doublet fields H_1 and H_2. Taking this into account, and using the minimisation conditions, we find that the potential (4) has three types of stationary points:

1. The normal minimum with the following field configuration:

$$\phi_7 = v_1, \phi_8 = v_2, \phi_9 = v_3, \phi_i = 0, \quad i \neq 7, 8, 9$$

2. The stationary point which breaks electric charge:

$$\phi_7 = v_1', \phi_8 = v_2', \phi_9 = v_3', \phi_1 = \alpha_1, \phi_2 = \alpha_2, \phi_3 = \alpha_3,$$

3. The CP breaking minimum.

$$\phi_7 = v_1'', \phi_8 = v_2'', \phi_9 = v_3'', \phi_{10} = \gamma_1, \phi_{11} = \gamma_2, \phi_{12} = \gamma_3.$$

The CP Breaking minimum

The CP breaking minimum (CPB) [3] we have

$$\langle\Phi_i\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_i + i\gamma_i \end{pmatrix} \qquad i = 1, 2, 3\,, \tag{6}$$

where $\gamma_i \in \Re$. Then, CPB is at

$$\begin{gathered} \phi_7 = v_1,\ \phi_8 = v_2,\ \phi_9 = v_3,\ \phi_{10} = \gamma_1,\ \phi_{11} = \gamma_2,\ \phi_{12} = \gamma_3, \\ \text{and other cases} \qquad \phi_i = 0\,, \end{gathered} \tag{7}$$

which should satisfy the constraint

$$v = \left(v_1^2 + v_2^2 + v_3^2 + \gamma_1^2 + \gamma_2^2 + \gamma_3^2\right)^{1/2}. \tag{8}$$

To complete the story, the constants γ_i can take the values following:

- $\gamma_1 \neq 0$, and $\gamma_2 = \gamma_3 = 0$;
- $\gamma_2 \neq 0$, and $\gamma_1 = \gamma_3 = 0$;
- $\gamma_3 \neq 0$, and $\gamma_1 = \gamma_2 = 0$;
- $\gamma_1 \neq 0$, $\gamma_2 \neq 0$, and $\gamma_3 = 0$;
- $\gamma_1 \neq 0$, $\gamma_3 \neq 0$, and $\gamma_2 = 0$;
- $\gamma_2 \neq 0$, $\gamma_3 \neq 0$, and $\gamma_1 = 0$; and
- $\gamma_1 \neq 0$, $\gamma_2 \neq 0$, and $\gamma_3 \neq 0$.

We assume the Higgs vev's are free parameters subject to the constraint (8).

The fermionic mass matrices

The most general S_3 invariant Yukawa Lagrangian with three Higgs doublets [6] can be written as

$$\mathcal{L}_Y = \mathcal{L}_{Y_D} + \mathcal{L}_{Y_u} + \mathcal{L}_{Y_E} + \mathcal{L}_{Y_\nu}, \tag{9}$$

Each term is given for down and up family as

$$\begin{aligned}
\mathcal{L}_{Y_D} &= -Y_1^d \overline{Q}_I H_S d_{IR} - Y_3^d \overline{Q}_3 H_S d_{3R} \\
&- Y_2^d [\, \overline{Q}_I \kappa_{IJ} H_1 d_{JR} + \overline{Q}_I \eta_{IJ} H_2 d_{JR} \,] \\
&- Y_4^d \overline{Q}_3 H_I d_{IR} - Y_5^d \overline{Q}_I H_I d_{3R} + h.c, \qquad (10) \\
\mathcal{L}_{Y_U} &= -Y_1^u \overline{Q}_I (i\sigma_2) H_S^* u_{IR} - Y_3^u \overline{Q}_3 (i\sigma_2) H_S^* u_{3R} \\
&- Y_2^u [\, \overline{Q}_I \kappa_{IJ} (i\sigma_2) H_1^* u_{JR} + \eta \overline{Q}_I \eta_{IJ} (i\sigma_2) H_2^* u_{JR} \,] \\
&- Y_4^u \overline{Q}_3 (i\sigma_2) H_I^* u_{IR} - Y_5^u \overline{Q}_I (i\sigma_2) H_I^* u_{3R} + h.c., \qquad (11)
\end{aligned}$$

Singlets carry the index s or 3 and doublets carry indices $I, J = 1, 2$

$$\kappa = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{and} \quad \eta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{12}$$

From this, we can express the fermionic mass matrix $\mathbf{M}_f$ including spontaneous CP violation as

$$\mathbf{M}_f = \begin{pmatrix} m_1 + m_6 & m_2 & m_5 \\ m_2 & m_1 - m_6 & m_8 \\ m_4 & m_7 & m_3 \end{pmatrix}, \tag{13}$$

where

$$\begin{aligned}
m_1^d &= -Y_1^d\,(\phi_9 + i\phi_{12})\,, && (14)\\
m_2^d &= -Y_2^d\,(\phi_7 + i\phi_{10})\,, && (15)\\
m_3^d &= -Y_3^d\,(\phi_9 + i\phi_{12})\,, && (16)\\
m_4^d &= -Y_4^d\,(\phi_7 + i\phi_{10})\,, && (17)\\
m_5^d &= -Y_5^d\,(\phi_7 + i\phi_{10})\,, && (18)\\
m_6^d &= -Y_2^d\,(\phi_8 + i\phi_{11})\,, && (19)\\
m_7^d &= -Y_4^d\,(\phi_8 + i\phi_{11})\,, && (20)\\
m_8^d &= -Y_5^d\,(\phi_8 + i\phi_{11})\,. && (21)
\end{aligned}$$

Then, the fermionic mass matrices are complex caused by contribution arising from the Higgs sector. Thus, the SSB mechanism provides a source for CP violation in the fermionic sector and contributes to the same in the quark and lepton mixing matrices.

CP violation

All CP violation, in the Electroweak Model, originates from the term which describes the interactions of the fermions with the higgs doublet.

Jarlskog invariant

These conditions must be satisfied in order to have CP violation [?]

$$\begin{aligned} m_u &\neq m_c,\ m_c \neq m_t,\ m_t \neq m_u,\ m_d \neq m_s,\ m_s \neq m_b,\ m_b \neq m_d, \\ \theta_j &\neq 0, \frac{\pi}{2}, \quad \delta \neq 0, \pi, \quad j = 1,2,3 \end{aligned} \tag{22}$$

where m_i are respective quark mass value. These conditions are unified within the single relation

$$\det C \neq 0 \tag{23}$$

where

$$iC = \left[M^u M^{u\dagger}, M^d M^{d\dagger} \right] \tag{24}$$

Jarlskog invariant

What is highly remarkable about the adove commutator is that determinant is given by

$$\begin{aligned} \det C &= -2J\left(m_t^2 - m_c^2\right)\left(m_c^2 - m_u^2\right)\left(m_u^2 - m_t^2\right) \\ &\quad \times \left(m_b^2 - m_s^2\right)\left(m_s^2 - m_d^2\right)\left(m_d^2 - m_b^2\right) \end{aligned} \tag{25}$$

where J is the Jarlskog invariant, related to CP violation.

Minimum conditions

The minimization conditions give us six equations determined by demanding of $\partial V/\partial\phi_i\mid_{min}= 0$.

Scenario	ϕ_7	ϕ_8	ϕ_9	ϕ_{10}	ϕ_{11}	ϕ_{12}
I	v_1	v_2	v_3	γ_1	γ_2	γ_3
II	v_1	v_2	v_3	γ_1	0	0
III	v_1	v_2	v_3	0	γ_2	0
IV	v_1	v_2	v_3	0	0	γ_3
V	v_1	v_2	v_3	γ_1	γ_2	0
VI	v_1	v_2	v_3	γ_1	0	γ_3
VII	v_1	v_2	v_3	0	γ_2	γ_3

anothers fields are nulls. Any scenario the minimum conditions have more variables than independent equations, we focus in VEVs of Higgs to use the commutator formalism of Jarlskog.

Scenario I

The minimum conditions in this scenario are :

$$
\begin{aligned}
0 &= 2\,(c+g)\,{v_1}^3 + 2\,\gamma_1\,(v_2\,(2\,(d+g)\,\gamma_2 + e\,\gamma_3) + v_3\,(e\,\gamma_2 + 2\,h\,\gamma_3)) \\
&+ v_1\,(2\,(c+g)\,{v_2}^2 + 6\,e\,v_2\,v_3 + (b+f+2\,h)\,{v_3}^2 + 2\,(c+g)\,{\gamma_1}^2 \\
&+ (b+f-2\,h)\,{\gamma_3}^2 + 2\,\gamma_2\,((c-2\,d-g)\,\gamma_2 + e\,\gamma_3) + {\mu_1}^2), \qquad (26) \\
0 &= 2\,(c+g)\,{v_2}^3 - 3\,e\,{v_2}^2\,v_3 + {v_1}^2\,(2\,(c+g)\,v_2 + 3\,e\,v_3) \\
&+ 2\,v_1\,\gamma_1\,(2\,(d+g)\,\gamma_2 + e\,\gamma_3) + v_3\,(e\,{\gamma_1}^2 + \gamma_2\,(-(e\,\gamma_2) + 4\,h\,\gamma_3)) \\
&+ v_2\,((b+f+2\,h)\,{v_3}^2 + 2\,(c-2\,d-g)\,{\gamma_1}^2 + 2\,(c+g)\,{\gamma_2}^2 \\
&- 2\,e\,\gamma_2\,\gamma_3 + (b+f-2\,h)\,{\gamma_3}^2 + {\mu_1}^2), \qquad (27) \\
0 &= -\left(e\,{v_2}^3\right) + (b+f+2\,h)\,{v_2}^2\,v_3 + {v_1}^2\,(3\,e\,v_2 + (b+f+2\,h)\,v_3) \\
&+ 2\,v_1\,\gamma_1\,(e\,\gamma_2 + 2\,h\,\gamma_3) + v_2\,\left(e\,{\gamma_1}^2 - e\,{\gamma_2}^2 + 4\,h\,\gamma_2\,\gamma_3\right) \\
&+ v_3\,\left((b+f-2\,h)\,\left({\gamma_1}^2 + {\gamma_2}^2\right) + 2\,a\,\left({v_3}^2 + {\gamma_3}^2\right) + {\mu_0}^2\right), \qquad (28)
\end{aligned}
$$

$$
\begin{aligned}
0 &= 2\,(c+g)\,{v_1}^2\,\gamma_1 + 2\,v_1\,\left(v_2\,\left(2\,(d+g)\,\gamma_2 + e\,\gamma_3\right) + v_3\,\left(e\,\gamma_2 + 2\,h\,\gamma_3\right)\right) \\
&\quad +\gamma_1\,(2\,(c-2\,d-g)\,{v_2}^2 + 2\,e\,v_2\,v_3 + (b+f-2\,h)\,{v_3}^2 \\
&\quad +2\,(c+g)\,\left({\gamma_1}^2 + {\gamma_2}^2\right) \\
&\quad +6\,e\,\gamma_2\,\gamma_3 + (b+f+2\,h)\,{\gamma_3}^2 + {\mu_1}^2), \qquad (29) \\
0 &= 2\,v_1\,\left(2\,(d+g)\,v_2 + e\,v_3\right)\,\gamma_1 + b\,{v_3}^2\,\gamma_2 + f\,{v_3}^2\,\gamma_2 \\
&\quad -2\,h\,{v_3}^2\,\gamma_2 + 2\,c\,{\gamma_1}^2\,\gamma_2 + 2\,g\,{\gamma_1}^2\,\gamma_2 \\
&\quad +2\,c\,{\gamma_2}^3 + 2\,g\,{\gamma_2}^3 + 3\,e\,{\gamma_1}^2\,\gamma_3 - 3\,e\,{\gamma_2}^2\,\gamma_3 + b\,\gamma_2\,{\gamma_3}^2 + f\,\gamma_2\,{\gamma_3}^2 + 2\,h\,\gamma_2\,{\gamma_3}^2 \\
&\quad +{v_2}^2\,\left(2\,(c+g)\,\gamma_2 - e\,\gamma_3\right) + {v_1}^2\,\left(2\,(c-2\,d-g)\,\gamma_2 + e\,\gamma_3\right) \\
&\quad -2\,v_2\,v_3\,\left(e\,\gamma_2 - 2\,h\,\gamma_3\right) + \gamma_2\,{\mu_1}^2, \qquad (30) \\
0 &= 2\,v_1\,\left(e\,v_2 + 2\,h\,v_3\right)\,\gamma_1 + 4\,h\,v_2\,v_3\,\gamma_2 + 3\,e\,{\gamma_1}^2\,\gamma_2 \\
&\quad -e\,{\gamma_2}^3 + 2\,a\,{v_3}^2\,\gamma_3 + b\,{\gamma_1}^2\,\gamma_3 + f\,{\gamma_1}^2\,\gamma_3 \\
&\quad +2\,h\,{\gamma_1}^2\,\gamma_3 + b\,{\gamma_2}^2\,\gamma_3 + f\,{\gamma_2}^2\,\gamma_3 + 2\,h\,{\gamma_2}^2\,\gamma_3 + 2\,a\,{\gamma_3}^3 \\
&\quad +{v_2}^2\,\left(-(e\,\gamma_2) + (b+f-2\,h)\,\gamma_3\right) \\
&\quad +{v_1}^2\,\left(e\,\gamma_2 + (b+f-2\,h)\,\gamma_3\right) + \gamma_3\,{\mu_0}^2. \qquad (31)
\end{aligned}
$$

Scenario I

We find many minimum conditions, we work with the jarlskog formalism each minimum situation. Then we have the following scenario and minimum conditions with CP violation.

scenario I.1

We find $e=0$, $v_2=\frac{\gamma_2 v_1}{\gamma_1}$ y $\gamma_3=-\frac{v_1 v_3}{\gamma_1}$.

$$\mu_0=\pm\frac{i}{\gamma_1}\left[\left(v_1^2+\gamma_1^2\right)\left(2av_3^2+(b+f-2h)\left(\gamma_1^2+\gamma_2^2\right)\right)\right]^{\frac{1}{2}}. \tag{32}$$

$$\mu_1=\pm\frac{i}{\gamma_1}\left[\left(v_1^2+\gamma_1^2\right)\left((b+f-2h)\,v_3^2+2\,(c+g)\left(\gamma_1^2+\gamma_2^2\right)\right)\right]^{\frac{1}{2}}. \tag{33}$$

In this situation with commutator formalism

$$det\left[M^u M^{u\dagger}, M^d M^{d\dagger}\right]=-\frac{i}{\gamma_1^{10}}\left[A\left(v_3^2\left(B+{Y^d}_2{Y^d}_4{Y^u}_2{Y^u}_4 C+D\right)+E\right)\right], \tag{34}$$

where

$$
\begin{aligned}
A &= 2\gamma_2\left(\gamma_1^2-3\gamma_2^2\right)^2\left(3\gamma_1^2-\gamma_2^2\right)v_3\left(\gamma_1^2+v_1^2\right)^6\left(\left({Y^d}_1{Y^d}_4-{Y^d}_3{Y^d}_5\right){Y^u}_2{Y^u}_4+\right.\\
B &= \left(Y^d\right)_2^2\left(Y^d\right)_4^2{Y^u}_1\left({Y^u}_1\left(\left(Y^u\right)_4^2+\left(Y^u\right)_5^2\right)-2{Y^u}_3{Y^u}_4{Y^u}_5\right)\\
C &= \left(Y^d\right)_5^2\left(\left(Y^u\right)_3^2-\left(Y^u\right)_1^2\right)+2{Y^d}_3{Y^d}_5\left({Y^u}_3{Y^u}_5-{Y^u}_1{Y^u}_4\right)\\
&\quad -\left(Y^d\right)_1^2\left(Y^u\right)_5^2+\left(Y^d\right)_3^2\left(Y^u\right)_5^2+2{Y^d}_1{Y^d}_4{Y^u}_1{Y^u}_4-2{Y^d}_1{Y^d}_4{Y^u}_3{Y^u}_5\\
D &= {Y^d}_1\left({Y^d}_1\left(\left(Y^d\right)_4^2+\left(Y^d\right)_5^2\right)-2{Y^d}_3{Y^d}_4{Y^d}_5\right)\left(Y^u\right)_2^2\left(Y^u\right)_4^2\\
E &= \left(\gamma_1^2+\gamma_2^2\right)\left({Y^d}_4{Y^u}_2-{Y^d}_2{Y^u}_4\right)\left({Y^d}_2{Y^u}_2+{Y^d}_4{Y^u}_4\right)\left({Y^d}_2{Y^d}_4\left(Y^u\right)_5^2-\left(\right.\right.
\end{aligned}
$$

We have CP violation in this situation.

Scenario I.2

Another situation of minimum is with $e=0$, $v_2=-\frac{(d+g)\gamma_1\, v_1+h\gamma_3\, v_3}{(d+g)\gamma_2}$, $\gamma_3=0$ and $v_1=0$.

$$\mu_0=\pm\left[-2av_3^2-(b+f-2h)\left(\gamma_1^2+\gamma_2^2\right)\right]^{\frac{1}{2}} . \tag{41}$$

$$\mu_1=\pm\left[-(b+f-2h)\,v_3^2-2\,(c+g)\left(\gamma_1^2+\gamma_2^2\right)\right]^{\frac{1}{2}} . \tag{42}$$

In this situation with commutator formalism

$$det\left[M^u M^{u\dagger},M^d M^{d\dagger}\right]=-\frac{i}{\gamma_1^{10}}\left[F\left(v_3^2\left(B+{Y^d}_2{Y^d}_4{Y^u}_2{Y^u}_4C+D\right)+E\right)\right], \tag{43}$$

$$F=2\left(\gamma_1^3-3\gamma_1\gamma_2^2\right)^2\left(3\gamma_1^2\gamma_2-\gamma_2^3\right)v_3\left(\left({Y^d}_3{Y^d}_5-{Y^d}_1{Y^d}_4\right){Y^u}_2{Y^u}_4+{Y^d}_2{Y^d}_4\,({Y^u}\ldots\right.$$

. We have CP violation in this situation.

scenario I.3

In this situation we have $e = 0$, $v_2 = \frac{\gamma_2 \, v_1}{\gamma_1}$, $h = 0$

$$\mu_0 = \pm \frac{i}{\gamma_1} \left[(b + f) \left(v_1^2 + \gamma_1^2 \right) \left(\gamma_1^2 + \gamma_2^2 \right) + 2a\gamma_1^2 \left(v_3^2 + \gamma_3^2 \right) \right]^{\frac{1}{2}} . \tag{44}$$

De la ecuación (26) encontramos para μ_1

$$\mu_1 = \pm \frac{1}{\gamma_1} \left[-2 \left(c + g \right) \left(v_1^2 + \gamma_1^2 \right) \left(\gamma_1^2 + \gamma_2^2 \right) - \left(b + f \right) \gamma_1^2 \left(v_3^2 + \gamma_3^2 \right) \right]^{\frac{1}{2}} . \tag{45}$$

In this situation we have a result very extensive to express in this presentation, then we give values each yukawa parameter

scenario I.3

if $Y_4^d = 2$,, and another yukawa parameter are unity, we have:

$$det\left[M^u M^{u\dagger}, M^d M^{d\dagger}\right] = \frac{i}{\gamma_1^9} AB, \tag{46}$$

where

$$\begin{aligned}
A &= 2\left(\gamma_1^2 - 3\gamma_2^2\right)^2 \left(\gamma_1^2 + v_1^2\right)\left(\gamma_3 v_1 - \gamma_1 v_3\right)\left(3\left(\gamma_1^2 + \gamma_2^2\right) v_1^2 + \gamma_1^2 \left(\gamma_3^2 + 3\left(\gamma_1^2 + \gamma_2^2\right)\right.\right.\\
B &= \gamma_1^6\left(-\gamma_2^3 + 4\gamma_3\gamma_2^2 - 8\gamma_3^3 + \gamma_1^2\left(3\gamma_2 + 4\gamma_3\right)\right) + \left(3\gamma_1^2\gamma_2 - \gamma_2^3\right) v_1^6 + 4\gamma_1\left(\gamma_1^2 + \gamma_2^2\right)\\
&+\gamma_1^2\left(\left(9\gamma_2 + 4\gamma_3\right)\gamma_1^2 + \gamma_2^2\left(4\gamma_3 - 3\gamma_2\right)\right) v_1^4 + 8\gamma_1^3 v_3 v_1^3\left(\gamma_1^2 + \gamma_2^2 - v_3^2\right)\\
&+\gamma_1^4 v_1^2\left(-3\gamma_2^3 + 9\gamma_1^2\gamma_2 + 8\gamma_3\left(\gamma_1^2 + \gamma_2^2 - 3v_3^2\right)\right) + 4\gamma_1^5\left(\gamma_1^2 + \gamma_2^2 - 6\gamma_3^2\right) v_3 v_1
\end{aligned}$$

We have CP violation in this situation.

Minimum scenarios

Thus, in this situations presents CP violation

Scenario	conditions
I.1	$e=0,\ v_2=\frac{\gamma_2\, v_1}{\gamma_1}\ \gamma_3=-\frac{v_1 v_3}{\gamma_1}$
I.2	$e=0,\ v_2=-\frac{(d+g)\gamma_1\, v_1+h\gamma_3\, v_3}{(d+g)\gamma_2},\ \gamma_3=0,\ v_1=0$
I.3	$e=0,\ v_2=\frac{\gamma_2\, v_1}{\gamma_1},\ h=0$
II.1	$v_3=-\frac{2(d+g)}{e}v_2,\ e=\pm 2\sqrt{(d+g)\, h},\ v_2=\pm\sqrt{\frac{v_1^2+\gamma_1^2}{3}}$
III.1	$v_3=-\frac{2(d+g)}{e}v_2,\ v_1=\pm\sqrt{3v_2^2+\gamma_2^2},\ h=\frac{e^2}{4(d+g)}$
III.2	$v_3=-\frac{2v_2(d+g)}{e},\ e=\frac{2i\sqrt{2}\sqrt{h}v_2(d+g)}{\sqrt{(d+g)\left(\gamma_2^2+v_2^2\right)}},\ v_1=0$

Scenario	conditions
IV.1	$e=0,\ h=0,\ v_2=v_1$
IV.2	$e=0,\ v_3=0,\ v_2=v_1$
V.1	$e=0,\ h=0,\ v_2=\frac{\gamma_2}{\gamma_1}v_1$
V.2	$e=0,\ v_2=-v_1,\ \gamma_2=\gamma_1,\ h=-\frac{2(d+g)(v_1-\gamma_1)(\gamma_1+v_1)}{v_3^2}$
V.3	$e=0,\ v_2=-v_1,\ \gamma_2=\gamma_1,\ v_3=\pm\frac{i\sqrt{2}\sqrt{d+g}\sqrt{(v_1-\gamma_1)(\gamma_1+v_1)}}{\sqrt{h}}$
VI.1	$e=0,\ h=0,\ v_2=0$
VI.2	$e=0,\ d=-g,\ h=0,\ v_2=\gamma_1$
VI.3	$e=0,\ h=0,\ v_2=\gamma_1,\ v1=0$
VI.4	$e=0,\ h=0,\ v_2=\gamma_1,\ \gamma_1=0$
VII.1	$v_1=\sqrt{3}\sqrt{\gamma_2^2+v_2^2},\ v_3=\frac{\gamma_3 v_2}{\gamma_2},\ \gamma_3=-\frac{\gamma_2 e}{2h},\ e=2\sqrt{h(d+g)}$

the values of μ_0 for each scenario:

Scenario	μ_0
I.1	$\frac{i\sqrt{\gamma_1^2+v_1^2}\sqrt{2av_3^2+\left(\gamma_1^2+\gamma_2^2\right)(b+f-2h)}}{\gamma_1}$
I.2	$\sqrt{-2av_3^2-\left(\gamma_1^2+\gamma_2^2\right)(b+f-2h)}$
I.3	$\frac{i\sqrt{2a\gamma_1^2\left(\gamma_3^2+v_3^2\right)+b\left(\gamma_1^2+\gamma_2^2\right)\left(\gamma_1^2+v_1^2\right)+\left(\gamma_1^2+\gamma_2^2\right)f\left(\gamma_1^2+v_1^2\right)}}{\gamma_1}$
II.1	$\sqrt{\frac{2}{3}}\sqrt{-\frac{\left(\gamma_1^2+v_1^2\right)(a(d+g)+2h(b+f-2h))}{h}}$
III.1	$\frac{\sqrt{e^2\left(\gamma_2^2+2v_2^2\right)\left(e^2-2(b+f)(d+g)\right)-8av_2^2(d+g)^3}}{\sqrt{e^2(d+g)}}$
III.2	$\frac{(-1)^{3/4}\sqrt[4]{h}\sqrt{\frac{i\left((d+g)\left(\gamma_2^2+v_2^2\right)\right)^{3/2}(a(d+g)-h(b+f-2h))}{h^{3/2}(d+g)}}}{\sqrt[4]{(d+g)\left(\gamma_2^2+v_2^2\right)}}$
IV.1	$\sqrt{2}\sqrt{v_1^2(-b-f)-a\left(\gamma_3^2+v_3^2\right)}$
IV.2	$\sqrt{-2a\gamma_3^2-2v_1^2(b+f-2h)}$

Scenario	μ_0
V.1	$\frac{i\sqrt{2a\gamma_1^2 v_3^2+b\left(\gamma_1^2+\gamma_2^2\right)\left(\gamma_1^2+v_1^2\right)+\left(\gamma_1^2+\gamma_2^2\right)\left(\gamma_1^2+v_1^2\right)}}{\gamma_1}$
V.2	$\frac{\sqrt{v_3^2\left(\gamma_1^2(-b-f)\left(\gamma_1^2+v_1^2\right)-\gamma_1^2\left(2av_3^2+(b+f)\left(\gamma_1^2+v_1^2\right)\right)\right)+8\gamma_1^2 d\left(v_1^2-\gamma_1^2\right)^2+8\gamma_1^2 g\left(v_1^2-\gamma_1^2\right)^2}}{\gamma_1 v_3}$
V.3	$\sqrt{\frac{4a\gamma_1^2(d+g)(v_1-\gamma_1)(\gamma_1+v_1)-2\gamma_1^2 h\left(\gamma_1^2(b+f-2h)+v_1^2(b+f+2h)\right)}{\gamma_1^2 h}}$
VI.1	$\sqrt{(-b-f)\left(\gamma_1^2+v_1^2\right)-2a\left(\gamma_3^2+v_3^2\right)}$
VI.2	$\sqrt{(-b-f)\left(2\gamma_1^2+v_1^2\right)-2a\left(\gamma_3^2+v_3^2\right)}$
VI.3	$\sqrt{2}\sqrt{\gamma_1^2(-b-f)-a\left(\gamma_3^2+v_3^2\right)}$
VI.4	$\sqrt{v_1^2(-b-f)-2a\left(\gamma_3^2+v_3^2\right)}$
VII	$\frac{i\sqrt{2}\sqrt{\gamma_2^2+v_2^2}\sqrt{a(d+g)+2h(b+f-2h)}}{\sqrt{h}}$

the values of μ_1 for each scenario:

Scenario	μ_1
I.1	$\frac{i\sqrt{\gamma_1^2+v_1^2}\sqrt{v_3^2(b+f-2h)+2\left(\gamma_1^2+\gamma_2^2\right)(c+g)}}{\gamma_1}$
I.2	$\sqrt{v_3^2(-b-f+2h)-2\left(\gamma_1^2+\gamma_2^2\right)(c+g)}$
I.3	$\frac{\sqrt{\gamma_1^2(-(b+f))\left(\gamma_3^2+v_3^2\right)-2c\left(\gamma_1^2+\gamma_2^2\right)\left(\gamma_1^2+v_1^2\right)-2\left(\gamma_1^2+\gamma_2^2\right)g\left(\gamma_1^2+v_1^2\right)}}{\gamma_1}$
II.1	$\frac{\sqrt{-\frac{\left(\gamma_1^2+v_1^2\right)((b+f)(d+g)-2h(-4c+5d+g))}{h}}}{\sqrt{3}}$
III.1	$\frac{\sqrt{4\gamma_2^2e^2(d-c)-2v_2^2\left(g\left(4d(b+f)-e^2\right)+d\left(2d(b+f)-5e^2\right)+2g^2(b+f)+4ce^2\right)}}{e}$
III.2	$\frac{\sqrt{\gamma_2^2((b+f)(d+g)-2h(2c+d+3g))+v_2^2((b+f)(d+g)-2h(2c+5d+7g))}}{\sqrt{2}\sqrt{h}}$
IV.1	$\sqrt{-(b+f)\left(\gamma_3^2+v_3^2\right)-4v_1^2(c+g)}$
IV.2	$\sqrt{-\gamma_3^2(b+f-2h)-4v_1^2(c+g)}$

Scenario	μ_1
V.1	$\frac{\sqrt{\gamma_1^2 v_3^2(-b-f)-2c\left(\gamma_1^2+\gamma_2^2\right)\left(\gamma_1^2+v_1^2\right)-2\left(\gamma_1^2+\gamma_2^2\right)g\left(\gamma_1^2+v_1^2\right)}}{\gamma_1}$
V.2	$\frac{\sqrt{\gamma_1^2\left(v_3^2(-b-f)-2\left(\gamma_1^2+v_1^2\right)(c-2d-g)\right)-2\gamma_1^2(c+g)\left(\gamma_1^2+v_1^2\right)}}{\gamma_1}$
V.3	$\frac{\sqrt{-2\gamma_1^4((b+f)(d+g)+h(c-2d-g))+2\gamma_1^2 v_1^2((b+f)(d+g)+h(-c+2d+g))-2\gamma_1^2 h(c+g)\left(\gamma_1^2+\right.}}{\gamma_1\sqrt{h}}$
VI.1	$\sqrt{-(b+f)\left(\gamma_3^2+v_3^2\right)-2(c+g)\left(\gamma_1^2+v_1^2\right)}$
VI.2	$\sqrt{-(b+f)\left(\gamma_3^2+v_3^2\right)-2(c+g)\left(2\gamma_1^2+v_1^2\right)}$
VI.3	$\sqrt{4\gamma_1^2(d-c)-(b+f)\left(\gamma_3^2+v_3^2\right)}$
VI.4	$\sqrt{-(b+f)\left(\gamma_3^2+v_3^2\right)-2v_1^2(c+g)}$
VII	$\frac{i\sqrt{\gamma_2^2+v_2^2}\sqrt{b(d+g)+8ch+f(d+g)-2h(5d+g)}}{\sqrt{h}}$

Summary and conclusions

In this work, we analyzed the SSB of SU(2) $\times$ U(1) $\rightarrow$ U(1)$_{em}$ in $S(3)$SM with spontaneous CPV provided by the Higgs sector. In this model, we introduced three Higgs SU(2) doublets with twelve real fields. While defining the gauge symmetry spontaneous breaking in eq. (7), we found a parameter space region where the minimum of the potential defines a CPB ground state. We analyzed seven possible scenarios defined in concordance with the CPV source Higgs field.

Thanks

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references

Minimum scenarios

Scenario I.- CP violation come from the three fields of Higgs:

$$\phi_7 = v_1, \phi_8 = v_2, \phi_9 = v_3, \phi_{10} = \gamma_1, \phi_{11} = \gamma_2, \phi_{12} = \gamma_3,$$

anothers fields are nulls. Scenario II.- CP violation come from H_1:

$$\phi_7 = v_1, \phi_8 = v_2, \phi_9 = v_3, \phi_{10} = \gamma_1,$$

anothers fields are nulls. Scenario III.- CP violation come from H_2:

$$\phi_7 = v_1, \phi_8 = v_2, \phi_9 = v_3, \phi_{11} = \gamma_2,$$

anothers fields are nulls.
Scenario IV.- CP violation come from H_s:

$$\phi_7 = v_1, \phi_8 = v_2, \phi_9 = v_3, \phi_{12} = \gamma_3,$$

anothers fields are nulls.

Scenario V.- CP violation come from the doublet of Higgs in S_3:

$$\phi_7 = v_1, \phi_8 = v_2, \phi_9 = v_3, \phi_{10} = \gamma_1, \phi_{11} = \gamma_2,$$

anothers fields are nulls. Scenario VI.- CP violation come from H_1 and H_s:

$$\phi_7 = v_1, \phi_8 = v_2, \phi_9 = v_3, \phi_{10} = \gamma_1, \phi_{12} = \gamma_3,$$

anothers fields are nulls. Scenario VII.- CP violation come from H_2 and H_s:

$$\phi_7 = v_1, \phi_8 = v_2, \phi_9 = v_3, \phi_{11} = \gamma_2, \phi_{12} = \gamma_3,$$

anothers fields are nulls.
Any scenario the minimum conditions have more variables than independent equations, we focus in VEVs of Higgs to use the commutator formalism of Jarlskog.

Scenario II

The minimum conditions in this scenario are ::

$$
\begin{aligned}
0 &= v_1\,(2\,(c+g)\,{v_1}^2 + 2\,(c+g)\,{v_2}^2 + 6\,e\,v_2\,v_3 + (b+f+2\,h)\,{v_3}^2 \\
&\quad + 2\,(c+g)\,{\gamma_1}^2 + {\mu_1}^2), && (51)\\
0 &= 2\,(c+g)\,{v_2}^3 - 3\,e\,{v_2}^2\,v_3 + {v_1}^2\,(2\,(c+g)\,v_2 + 3\,e\,v_3) + e\,v_3\,{\gamma_1}^2 \\
&\quad + v_2\,\left((b+f+2\,h)\,{v_3}^2 + 2\,(c-2\,d-g)\,{\gamma_1}^2 + {\mu_1}^2\right), && (52)\\
0 &= -\left(e\,{v_2}^3\right) + (b+f+2\,h)\,{v_2}^2\,v_3 + {v_1}^2\,(3\,e\,v_2 + (b+f+2\,h)\,v_3) \\
&\quad + e\,v_2\,{\gamma_1}^2 + v_3\,\left(2\,a\,{v_3}^2 + (b+f-2\,h)\,{\gamma_1}^2 + {\mu_0}^2\right), && (53)\\
0 &= \gamma_1\,(2\,(c+g)\,{v_1}^2 + 2\,(c-2\,d-g)\,{v_2}^2 + 2\,e\,v_2\,v_3 + (b+f-2\,h)\,{v_3}^2 \\
&\quad + 2\,(c+g)\,{\gamma_1}^2 + {\mu_1}^2), && (54)\\
0 &= v_1\,(2\,(d+g)\,v_2 + e\,v_3)\,\gamma_1, && (55)\\
0 &= v_1\,(e\,v_2 + 2\,h\,v_3)\,\gamma_1. && (56)
\end{aligned}
$$

scenario II.1

We find the following conditions

$$\begin{aligned} v_3 &= -\frac{2\,(d+g)}{e} v_2, \\ e &= -2\sqrt{(d+g)\,h}, \\ v_2 &= \sqrt{\frac{v_1^2+\gamma_1^2}{3}}, \end{aligned} \tag{57}$$

$$\mu_0 = \pm\left[-\frac{2\,(a\,(d+g)+2\,(b+f-2h)\,h)\left(v_1^2+\gamma_1^2\right)}{3h}\right]^{\frac{1}{2}} \tag{58}$$

$$\mu_1 = \pm\left[-\frac{((b+f)\,(d+g)-2\,(-4c+5d+g)\,h)\left(v_1^2+\gamma_1^2\right)}{3h}\right]^{\frac{1}{2}}. \tag{59}$$

scenario II.1

for yukawa parameters are of unity value, we have

$$det\left[M^u M^{u\dagger}, M^d M^{d\dagger}\right] = -\frac{64i\gamma_1^5 v_1 (e-4G)(e-2G)^2\left(\gamma_1^2+v_1^2\right)^3}{27e^3}, \qquad (60)$$

where $G = d + g$, we have CP violation in this situation.

scenario II.2

we find

$$\begin{aligned} v_3 &= -\frac{2\left(d+g\right)}{e} v_2, \\ e &= 2\sqrt{\left(d+g\right)h}, \\ v_2 &= \sqrt{\frac{v_1^2+\gamma_1^2}{3}}, \end{aligned} \tag{61}$$

$$\mu_0 = \pm\left[-\frac{2\left(a\left(d+g\right)+2\left(b+f-2h\right)h\right)\left(v_1^2+\gamma_1^2\right)}{3h}\right]^{\frac{1}{2}} \tag{62}$$

$$\mu_1 = \pm\left[-\frac{\left(\left(b+f\right)\left(d+g\right)-2\left(-4c+5d+g\right)h\right)\left(v_1^2+\gamma_1^2\right)}{3h}\right]^{\frac{1}{2}}. \tag{63}$$

scenario II.2

for yukawa parameters are of unity value, we have:

$$det\left[M^u M^{u\dagger}, M^d M^{d\dagger}\right] = -\frac{64i\gamma_1^5 v_1(e-4G)(e-2G)^2\left(\gamma_1^2+v_1^2\right)^3}{27e^3}. \qquad (64)$$

we have CP violation in this situation.

scenario II.3

we find:

$$
\begin{aligned}
e &= -\frac{d+g}{v_3} v_2 - \frac{h}{v_2} v_3, \\
v_2 &= -\sqrt{\frac{h}{d+g}} v_3, \\
v_3 &= -\sqrt{\frac{(d+g)\left(v_1^2+\gamma_1^2\right)}{3h}}.
\end{aligned} \tag{65}
$$

$$
\mu_0 = \pm\left[-\frac{2\left(a\left(d+g\right)+2\left(b+f-2h\right)h\right)\left(v_1^2+\gamma_1^2\right)}{3h}\right]^{\frac{1}{2}}, \tag{66}
$$

$$
\mu_1 = \pm i\left[-\frac{\left(\left(b+f\right)\left(d+g\right)-2\left(-4c+5d+g\right)h\right)\left(v_1^2+\gamma_1^2\right)}{3h}\right]^{\frac{1}{2}}. \tag{67}
$$

scenario II.3

for yukawa parameters are of unity value, we have:

$$det\left[M^u M^{u\dagger}, M^d M^{d\dagger}\right] = \frac{64i\gamma_1^5 v_1\left(\sqrt{G}-\sqrt{h}\right)^2\left(2\sqrt{G}-\sqrt{h}\right)\left(\gamma_1^2+v_1^2\right)^3}{27h^{3/2}}. \tag{68}$$

we have CP violation in this situation.

scenario II.4

we find:

$$\begin{aligned} e &= -\frac{d+g}{v_3} v_2 - \frac{h}{v_2} v_3, \\ v_2 &= \sqrt{\frac{h}{d+g}} v_3, \\ v_3 &= \sqrt{\frac{(d+g)\left(v_1^2+\gamma_1^2\right)}{3h}}. \end{aligned} \tag{69}$$

$$\mu_0 = \pm\left[-\frac{2\left(a\left(d+g\right)+2\left(b+f-2h\right)h\right)\left(v_1^2+\gamma_1^2\right)}{3h}\right]^{\frac{1}{2}} \tag{70}$$

$$\mu_1 = \pm i\left[-\frac{\left(\left(b+f\right)\left(d+g\right)-2\left(-4c+5d+g\right)h\right)\left(v_1^2+\gamma_1^2\right)}{3h}\right]^{\frac{1}{2}}. \tag{71}$$

Escenario II.4

for yukawa parameters are of unity value, we have:

$$det\left[M^{u}M^{u\dagger},M^{d}M^{d\dagger}\right]=\frac{64i\gamma_1^5v_1\left(\sqrt{G}-\sqrt{h}\right)^2\left(2\sqrt{G}-\sqrt{h}\right)\left(\gamma_1^2+v_1^2\right)^3}{27h^{3/2}}, \tag{72}$$

we have CP violation in this situation.

Scenario III

The minimum conditions in this scenario are :

$$
\begin{aligned}
0 &= v_1 \left(2\,(c+g)\,{v_1}^2 + 2\,(c+g)\,{v_2}^2 + 6\,e\,v_2\,v_3 + (b+f+2\,h)\,{v_3}^2\right. \\
&\quad \left. +2\,(c-2\,d-g)\,{\gamma_2}^2 + {\mu_1}^2\right), \qquad (73) \\
0 &= 2\,(c+g)\,{v_2}^3 - 3\,e\,{v_2}^2\,v_3 + {v_1}^2\,(2\,(c+g)\,v_2 + 3\,e\,v_3) - e\,v_3\,{\gamma_2}^2 \\
&\quad +v_2\,\left((b+f+2\,h)\,{v_3}^2 + 2\,(c+g)\,{\gamma_2}^2 + {\mu_1}^2\right), \qquad (74) \\
0 &= -\left(e\,{v_2}^3\right) + (b+f+2\,h)\,{v_2}^2\,v_3 + {v_1}^2\,(3\,e\,v_2 + (b+f+2\,h)\,v_3) \\
&\quad -e\,v_2\,{\gamma_2}^2 + v_3\,\left(2\,a\,{v_3}^2 + (b+f-2\,h)\,{\gamma_2}^2 + {\mu_0}^2\right), \qquad (75) \\
0 &= v_1\,(2\,(d+g)\,v_2 + e\,v_3)\,\gamma_2, \qquad (76) \\
0 &= \gamma_2 \left(2\,(c-2\,d-g)\,{v_1}^2 + 2\,(c+g)\,{v_2}^2 - 2\,e\,v_2\,v_3 + (b+f-2\,h)\,{v_3}^2\right. \\
&\quad \left. +2\,(c+g)\,{\gamma_2}^2 + {\mu_1}^2\right), \qquad (77) \\
0 &= \gamma_2\,\left(-\left(e\,{v_1}^2\right) + e\,{v_2}^2 - 4\,h\,v_2\,v_3 + e\,{\gamma_2}^2\right). \qquad (78)
\end{aligned}
$$

Scenario III.1

we find

$$v_3 = -\frac{2\,(d+g)}{e} v_2, \tag{79}$$

$$v_1 = \pm\sqrt{3v_2^2 + \gamma_2^2}. \tag{80}$$

$$h = \frac{e^2}{4\,(d+g)}. \tag{81}$$

scenario III.1

$$\mu_0 = \pm\left[\frac{-8a\,(d+g)^3\,v_2^2+e^2\left(e^2-2\,(b+f)\,(d+g)\right)\left(2v_2^2+\gamma_2^2\right)}{e^2\,(d+g)}\right]^{\frac{1}{2}},$$

$$\mu_1 = \pm\left[\frac{-2\left(4ce^2+d\left(-5e^2+2d\,(b+f)\right)+\left(-e^2+4d\,(b+f)\right)g+2\,(b+f)\,g^2\right)}{e}\right.$$

give values of $Y_2^u = 2$ and $Y_i^{u,d} = 1$ to express in a short way the commutator formalism :

$$det\left[M^u M^{u\dagger}, M^d M^{d\dagger}\right] = \frac{i}{e^5}256\gamma_2^3 v_2^3(2e-9G)(e-2G)(e-G)\left(\gamma_2^2+3v_2^2\right)A, \tag{84}$$

where

$$A = \left(-7\gamma_2^4 e^2 - 2\gamma_2^2 v_2^2\left(9e^2+3eG+2G^2\right)+4v_2^4\left(3e^2-6eG-4G^2\right)\right)$$

we have CP violation.

Scenario IV

The minimum conditions in this scenario are :

$$
\begin{aligned}
0 &= v_1\,(2\,(c+g)\,{v_1}^2+2\,(c+g)\,{v_2}^2+6\,e\,v_2\,v_3+(b+f+2\,h)\,{v_3}^2 \\
&\quad +(b+f-2\,h)\,{\gamma_3}^2+{\mu_1}^2), && (85)\\
0 &= {v_1}^2\,(2\,(c+g)\,v_2+3\,e\,v_3)+v_2(2\,(c+g)\,{v_2}^2-3\,e\,v_2\,v_3 \\
&\quad +(b+f+2\,h)\,{v_3}^2+(b+f-2\,h)\,{\gamma_3}^2+{\mu_1}^2), && (86)\\
0 &= -\left(e\,{v_2}^3\right)+(b+f+2\,h)\,{v_2}^2\,v_3+{v_1}^2\,(3\,e\,v_2+(b+f+2\,h)\,v_3) \\
&\quad +v_3\,\left(2\,a\,\left({v_3}^2+{\gamma_3}^2\right)+{\mu_0}^2\right), && (87)\\
0 &= v_1\,(e\,v_2+2\,h\,v_3)\,\gamma_3, && (88)\\
0 &= \left(e\,{v_1}^2+v_2\,(-(e\,v_2)+4\,h\,v_3)\right)\,\gamma_3, && (89)\\
0 &= \gamma_3\,\left((b+f-2\,h)\,\left({v_1}^2+{v_2}^2\right)+2\,a\,\left({v_3}^2+{\gamma_3}^2\right)+{\mu_0}^2\right). && (90)
\end{aligned}
$$

scenario IV

We dont find any situation when the jarlskog invariante are not null. Maybe in future searchs we have situations with cp violation.

scenario V

The minimum conditions in this scenario are :

$$
\begin{aligned}
0 &= 2\,(c+g)\,v_1{}^3 + 2\,(2\,(d+g)\,v_2 + e\,v_3)\,\gamma_1\,\gamma_2 \\
&\quad + v_1\,(2\,(c+g)\,v_2{}^2 + 6\,e\,v_2\,v_3 + (b+f+2\,h)\,v_3{}^2 \\
&\quad + 2\,(c+g)\,\gamma_1{}^2 + 2\,(c-2\,d-g)\,\gamma_2{}^2 + \mu_1{}^2), \qquad (91)\\
0 &= 2\,(c+g)\,v_2{}^3 - 3\,e\,v_2{}^2\,v_3 + v_1{}^2\,(2\,(c+g)\,v_2 + 3\,e\,v_3) \\
&\quad + 4\,(d+g)\,v_1\,\gamma_1\,\gamma_2 + e\,v_3\,\left(\gamma_1{}^2 - \gamma_2{}^2\right) + v_2\,((b+f+2\,h)\,v_3{}^2 \\
&\quad + 2\,(c-2\,d-g)\,\gamma_1{}^2 + 2\,(c+g)\,\gamma_2{}^2 + \mu_1{}^2), \qquad (92)\\
0 &= -\left(e\,v_2{}^3\right) + (b+f+2\,h)\,v_2{}^2\,v_3 \\
&\quad + v_1{}^2\,(3\,e\,v_2 + (b+f+2\,h)\,v_3) + 2\,e\,v_1\,\gamma_1\,\gamma_2 \\
&\quad + e\,v_2\,\left(\gamma_1{}^2 - \gamma_2{}^2\right) + v_3\,\left(2\,a\,v_3{}^2 + (b+f-2\,h)\,\left(\gamma_1{}^2 + \gamma_2{}^2\right) + \mu_0{}^2\right) \qquad (93)
\end{aligned}
$$

Scenario V

$$\begin{aligned}
0 &= 2\,(c+g)\,{v_1}^2\gamma_1 + 2\,v_1\,(2\,(d+g)\,v_2 + e\,v_3)\,\gamma_2 \\
&\quad +\gamma_1\,(2\,(c-2\,d-g)\,{v_2}^2 + 2\,e\,v_2\,v_3 \\
&\quad +(b+f-2\,h)\,{v_3}^2 + 2\,(c+g)\,\left({\gamma_1}^2 + {\gamma_2}^2\right) + {\mu_1}^2), \qquad (94) \\
0 &= 2\,v_1\,(2\,(d+g)\,v_2 + e\,v_3)\,\gamma_1 + 2\,(c-2\,d-g)\,{v_1}^2\gamma_2 \\
&\quad +\gamma_2\,(2\,(c+g)\,{v_2}^2 - 2\,e\,v_2\,v_3 \\
&\quad +(b+f-2\,h)\,{v_3}^2 + 2\,(c+g)\,\left({\gamma_1}^2 + {\gamma_2}^2\right) + {\mu_1}^2), \qquad (95) \\
0 &= 4\,v_1\,(e\,v_2 + 2\,h\,v_3)\,\gamma_1 \\
&\quad +2\,\left(e\,{v_1}^2 - e\,{v_2}^2 + 4\,h\,v_2\,v_3 + 3\,e\,{\gamma_1}^2\right)\gamma_2 - 2\,e\,{\gamma_2}^3. \qquad (96)
\end{aligned}$$

scenario V.1

we find in the minimum conditions $e = 0$, $v_2 = \frac{\gamma_2}{\gamma_1} v_1$:

$$\mu_0 = \pm\frac{i\sqrt{2a\gamma_1^2 v_3^2 + b\left(\gamma_1^2+\gamma_2^2\right)\left(\gamma_1^2+v_1^2\right) + \left(\gamma_1^2+\gamma_2^2\right) f\left(\gamma_1^2+v_1^2\right)}}{\gamma_1}, \tag{97}$$

$$\mu_1 = \pm\frac{\sqrt{\gamma_1^2 v_3^2(-b-f) - 2c\left(\gamma_1^2+\gamma_2^2\right)\left(\gamma_1^2+v_1^2\right) - 2\left(\gamma_1^2+\gamma_2^2\right) g\left(\gamma_1^2+v_1^2\right)}}{\gamma_1} \tag{98}$$

scenario V.1

For $Y_1^u = 2$ and $Y_i^{u,d} = 1$ we have:

$$det\left[M^u M^{u\dagger}, M^d M^{d\dagger}\right] = -\frac{i}{\gamma_1^6} 2\left(\gamma_1^2 - 3\gamma_2^2\right)^2 v_3^3 \left(\gamma_1^2 + v_1^2\right) A, \qquad (99)$$

where

$$\begin{aligned} A &= 4\gamma_1^3 v_1 v_3 \left(\gamma_1^4 + 2v_1^2\left(\gamma_1^2 - v_3^2\right) + v_1^4\right) & (100) \\ &+ 3\gamma_2\gamma_1^2\left(\gamma_1^2 + v_1^2\right)^3 + 4\gamma_2^2\gamma_1 v_1 v_3\left(\gamma_1^2 + v_1^2\right)^2 - \gamma_2^3\left(\gamma_1^2 + v_1^2\right)^3 & (101) \end{aligned}$$

we have CP violation.

scenario VI

The minimum conditions in this scenario are :

$$
\begin{aligned}
0 &= 2\,(c+g)\,{v_1}^3 + 2\,(e\,v_2 + 2\,h\,v_3)\,\gamma_1\,\gamma_3 + v_1\,(2\,(c+g)\,{v_2}^2 + 6\,e\,v_2\,v_3 \\
&\quad + (b+f+2\,h)\,{v_3}^2 + 2\,(c+g)\,{\gamma_1}^2 + (b+f-2\,h)\,{\gamma_3}^2 + {\mu_1}^2), \qquad (102)\\
0 &= 2\,(c+g)\,{v_2}^3 - 3\,e\,{v_2}^2\,v_3 + {v_1}^2\,(2\,(c+g)\,v_2 + 3\,e\,v_3) + e\,v_3\,{\gamma_1}^2 + 2\,e\,v_1\,\gamma_1\,\gamma_3 \\
&\quad + v_2\,\left((b+f+2\,h)\,{v_3}^2 + 2\,(c-2\,d-g)\,{\gamma_1}^2 + (b+f-2\,h)\,{\gamma_3}^2 + {\mu_1}^2\right), \qquad (103)\\
0 &= -\left(e\,{v_2}^3\right) + (b+f+2\,h)\,{v_2}^2\,v_3 + {v_1}^2\,(3\,e\,v_2 + (b+f+2\,h)\,v_3) + e\,v_2\,{\gamma_1}^2 \\
&\quad + 4\,h\,v_1\,\gamma_1\,\gamma_3 + v_3\,\left((b+f-2\,h)\,{\gamma_1}^2 + 2\,a\,\left({v_3}^2 + {\gamma_3}^2\right) + {\mu_0}^2\right), \qquad (104)\\
0 &= 2\,(c+g)\,{v_1}^2\,\gamma_1 + 2\,v_1\,(e\,v_2 + 2\,h\,v_3)\,\gamma_3 + \gamma_1\,(2\,(c-2\,d-g)\,{v_2}^2 + 2\,e\,v_2\,v_3 \\
&\quad + (b+f-2\,h)\,{v_3}^2 + 2\,(c+g)\,{\gamma_1}^2 + (b+f+2\,h)\,{\gamma_3}^2 + {\mu_1}^2), \qquad (105)\\
0 &= 4\,v_1\,(2\,(d+g)\,v_2 + e\,v_3)\,\gamma_1 + 2\,\left(e\,{v_1}^2 - e\,{v_2}^2 + 4\,h\,v_2\,v_3 + 3\,e\,{\gamma_1}^2\right)\,\gamma_3, \qquad (106)\\
0 &= 2\,v_1\,(e\,v_2 + 2\,h\,v_3)\,\gamma_1 + (b+f-2\,h)\,{v_1}^2\,\gamma_3 \\
&\quad + \gamma_3\,\left((b+f-2\,h)\,{v_2}^2 + (b+f+2\,h)\,{\gamma_1}^2 + 2\,a\,\left({v_3}^2 + {\gamma_3}^2\right) + {\mu_0}^2\right). \qquad (107)
\end{aligned}
$$

scenario VI.1

we find $e = 0$ y $h = 0$, and $v_2 = 0$

$$\mu_0 = \pm\sqrt{(-b-f)\left(\gamma_1^2+v_1^2\right)-2a\left(\gamma_3^2+v_3^2\right)} \tag{108}$$

$$\mu_1 = \pm\sqrt{-(b+f)\left(\gamma_3^2+v_3^2\right)-2(c+g)\left(\gamma_1^2+v_1^2\right)}. \tag{109}$$

scenario VI.1

For $Y_1^u = 2$ and $Y_i^{u,d} = 1$ we have:

$$det\left[M^u M^{u\dagger}, M^d M^{d\dagger}\right] = 8i\left(\gamma_1^2 + v_1^2\right)\left(\gamma_3 v_1 - \gamma_1 v_3\right)\left(\gamma_1\gamma_3 + v_1 v_3\right)\left(\gamma_3^2 + v_3^2\right)\left(\gamma_1^4 - 2\right. \tag{110}$$

we have CP violation.

scenario VI.2

we find $e = 0$,$d = -g$, $h = 0$ y $v_2 = \gamma_1$

$$\mu_0 = \pm\sqrt{(-b-f)\left(2\gamma_1^2+v_1^2\right)-2a\left(\gamma_3^2+v_3^2\right)}, \quad (111)$$

$$\mu_1 = \pm\sqrt{-(b+f)\left(\gamma_3^2+v_3^2\right)-2(c+g)\left(2\gamma_1^2+v_1^2\right)}. \quad (112)$$

For $Y_i^{u,d} = 1$ we have:

$$det\left[M^u M^{u\dagger}, M^d M^{d\dagger}\right] = 16i\gamma_1^6\left(v_1^5\left(-\left(3\gamma_1+4v_3\right)\right)-4\gamma_1\gamma_3v_1^4+4v_1^3\left(\gamma_1^3+2v_3^3\right)+A\right) \quad (113)$$

where

$$A = 4\gamma_1\gamma_3v_1^2\left(4\gamma_1^2+5\gamma_1v_3+6v_3^2\right)-4\gamma_1^2v_1\left(5\gamma_1+6v_3\right)\left(v_3-\gamma_3\right)\left(\gamma_3+v_3\right)+8\gamma_1^3\gamma_3\left(\gamma_3^2 - \ldots\right.$$

we have CP violation.

scenario VI.3

We find $e = 0$, $h = 0$, $v_2 = \gamma_1$ and $v1 = 0$

$$\mu_0 = \pm\sqrt{2}\sqrt{\gamma_1^2(-b-f) - a\left(\gamma_3^2 + v_3^2\right)}, \quad (114)$$

$$\mu_1 = \pm\sqrt{4\gamma_1^2(d-c) - (b+f)\left(\gamma_3^2 + v_3^2\right)}. \quad (115)$$

For $Y_i^{u,d} = 1$ we have:

$$det\left[M^u M^{u\dagger}, M^d M^{d\dagger}\right] = 128 i \gamma_1^9 \gamma_3 \left(\gamma_3^2 - 3v_3^2\right), \quad (116)$$

we have CP violation.

scenario VI.4

we find $e = 0$, $h = 0$, $v_2 = \gamma_1$ and $\gamma_1 = 0$:

$$\mu_0 = \pm\sqrt{v_1^2(-b-f) - 2a\left(\gamma_3^2 + v_3^2\right)}, \tag{117}$$

$$\mu_1 = \pm\sqrt{-(b+f)\left(\gamma_3^2 + v_3^2\right) - 2v_1^2(c+g)}. \tag{118}$$

scenario VI.4

For $Y_1^u = 2$ and $Y_i^{u,d} = 1$ we have:

$$det\left[M^u M^{u\dagger}, M^d M^{d\dagger}\right] = 8i\gamma_3 v_1^6 v_3 \left(v_1^2 - 2v_3^2\right)\left(\gamma_3^2 + v_3^2\right), \qquad (119)$$

we have CP violation.

Scenario VII

The minimum conditions in this scenario are :

$$
\begin{aligned}
0 &= 2\,v_1\,(2\,(c+g)\,{v_1}^2 + 2\,(c+g)\,{v_2}^2 + 6\,e\,v_2\,v_3 + (b+f+2\,h)\,{v_3}^2 \\
&\quad + (b+f-2\,h)\,{\gamma_3}^2 + 2\,\gamma_2\,((c-2\,d-g)\,\gamma_2 + e\,\gamma_3) + {\mu_1}^2), \qquad (120)\\
0 &= 2\,(c+g)\,{v_2}^3 - 3\,e\,{v_2}^2\,v_3 + {v_1}^2\,(2\,(c+g)\,v_2 + 3\,e\,v_3) \\
&\quad + v_3\,\gamma_2\,(-(e\,\gamma_2) + 4\,h\,\gamma_3) \\
&\quad + v_2\,((b+f+2\,h)\,{v_3}^2 + (b+f-2\,h)\,{\gamma_3}^2 \\
&\quad + 2\,\gamma_2\,((c+g)\,\gamma_2 - e\,\gamma_3) + {\mu_1}^2,
\end{aligned}
$$

(121)

$$
\begin{aligned}
&0 = -\left(e\,{v_2}^3\right) + (b+f+2\,h)\,{v_2}^2\,v_3 + {v_1}^2\,(3\,e\,v_2 + (b+f+2\,h)\,v_3) \\
&+ v_2\,\gamma_2\,(-(e\,\gamma_2) + 4\,h\,\gamma_3) + v_3\,((b+f-2\,h)\,{\gamma_2}^2 \\
&+ 2\,a\left({v_3}^2 + {\gamma_3}^2\right) + {\mu_0}^2, \quad (122)
\end{aligned}
$$

Escenario VII

$$0 = v_1\ (v_2\ (2\ (d+g)\ \gamma_2 + e\,\gamma_3) + v_3\ (e\,\gamma_2 + 2\,h\,\gamma_3))\,, \quad (123)$$

$$\begin{aligned} 0 = {} & 2\,({v_2}^2\ (2\ (c+g)\ \gamma_2 - e\,\gamma_3) + {v_1}^2\ (2\ (c - 2\,d - g)\ \gamma_2 + e\,\gamma_3) \\ & -2\,v_2\,v_3\ (e\,\gamma_2 - 2\,h\,\gamma_3) \\ & +\gamma_2\ ((b+f-2\,h)\ {v_3}^2 + (b+f+2\,h)\ {\gamma_3}^2 \\ & +\gamma_2\ (2\ (c+g)\ \gamma_2 - 3\,e\,\gamma_3) + {\mu_1}^2), \end{aligned}$$

(124)

$$\begin{aligned} 0 = {} & 2\,(4\,\mathrm{h}\,\mathrm{v}_2\,v_3\,\gamma_2 - e\,{\gamma_2}^3 + 2\,a\,{v_3}^2\,\gamma_3 + b\,{\gamma_2}^2\,\gamma_3 \\ & +\mathrm{f}\,\gamma_2^2\,\gamma_3 + 2\,h\,{\gamma_2}^2\,\gamma_3 + 2\,a\,{\gamma_3}^3 \\ & +\mathrm{v}_2^2\ (-(e\,\gamma_2) + (b+f-2\,h)\ \gamma_3) \\ & +\mathrm{v}_1^2\ (e\,\gamma_2 + (b+f-2\,h)\ \gamma_3) + \gamma_3\,{\mu_0}^2).(125) \end{aligned}$$

Scenario VII

We dont find any situation when the jarlskog invariante are not null. Maybe in future searchs we have situations with cp violation.