Constrained Flavor Violation

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Completing constrained flavor violation: Lepton masses, neutrinos, and leptogenesis

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MINIMAL FLAVOR VIOLATION

- Minimal Flavor Violation (MFV) essentially requires that all flavor and CP-violating interactions are linked to the known structure of Yukawa couplings.
- The SM fermions consist of three families with two $SU(2)_L$ doublets (Q_L and L_L) and three $SU(2)_L$ singlets (U_R, D_R and E_R).
- The largest group of unitary field transformations that commutes with the gauge group is $U(3)^5$.

$$G_f = SU(3)_q^3 \times SU(3)_l^2 \times U(1)_B \times U(1)_L \times U(1)_Y \times U(1)_{PQ} \times U(1)_{E_R}$$

$$SU(3)_q^3 = SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R}$$

$$SU(3)_l^2 = SU(3)_{L_L} \times SU(3)_{E_R}$$

MINIMAL FLAVOR VIOLATION

- In the SM the Yukawa interactions break the symmetry group $SU(3)_q^3 \times SU(3)_l^2 \times U(1)_{PQ} \times U(1)_{E_R}$.
- We can recover flavor invariance by introducing dimensionless auxiliary fields Y_U,Y_D, and Y_E transforming under $SU(3)_q^3 \times SU(3)_l^2$ as:

$$Y_U \sim (3, \bar{3}, 1)_{SU(3)_q^3}, \qquad Y_D \sim (3, 1, \bar{3})_{SU(3)_q^3}, \qquad Y_E \sim (3, \bar{3})_{SU(3)_l^2}$$

- This allows the appearance of Yukawa interactions, consistently with the flavor symmetry

$$\mathcal{L} = \bar{Q}_L Y_D D_R H + \bar{Q}_L Y_U U_R H_c + \bar{L}_L Y_E E_R H + h.c.$$

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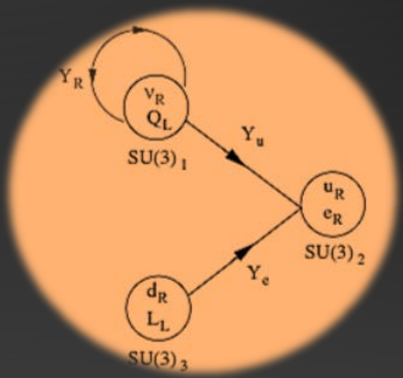
MOTIVATION

- A proposal that is a more predictive version of MFV in which there are only two fundamental Yukawa matrices, Y_e and Y_u, while the third one Y_u is predicted to be the product,

$$Y_d = \eta Y_u Y_e^\dagger$$

- This structure s a consequence of a spontaneously broken flavor symmetry $SU(3)_1 \times SU(3)_2 \times SU(3)_3$,



LEPTON MASS RATIOS

- It is argued that the charged lepton mass ratios are predicted almost correctly.
- The authors dub this scenario "constrained flavor breaking"; we called it "constrained flavor violation"(CFV) in analogy to MFV.
- The prediction of CFV for ratios of the charged lepton masses is a fundamental test since these arise directly from the prediction by solving for Y_e:

$$Ye = \eta^{-1}(Y_u^{-1}Y_d)^{\dagger} = \eta^{-1} diag(Y_d) V_{CKM}^{\dagger} diag(Y_u)^{-1}$$

LEPTON MASS RATIOS

- By inputting the measured quark masses and CKM mixings, within experimental uncertainties, one can generate Y_e, find its eigenvalues, and compute the mass ratios m_μ/m_τ, m_e/m_τ.
- In the previous work they carried this out for a large ensemble of randomly generated models, taking $1, 2$ and 3σ variations in the input parameters; only for a small fraction near the edge of the 3σ allowed region is m_μ/m_τ as large as its measured value.

LEPTON MASS RATIOS

- We adopted the running quark masses at the scale m_Z along with uncertainties and ranges given for the CKM matrix elements by the Particle Data Group (PDG).
- With these inputs, a scan over 3×10^5 models fails to produce any with $m_\mu/m_\tau > 0.045$ even at 3σ, whereas the observed value is close to 0.06.
- The discrepancy is worse than that found in the previous work due to the authors' larger and unexplained estimates of the experimental errors.

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Constrained flavor breaking

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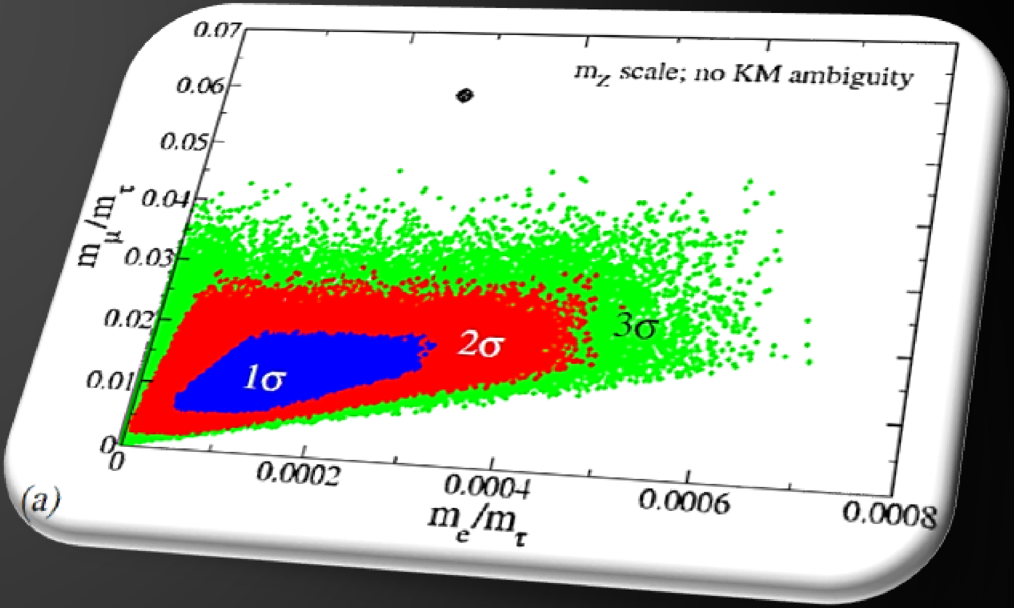

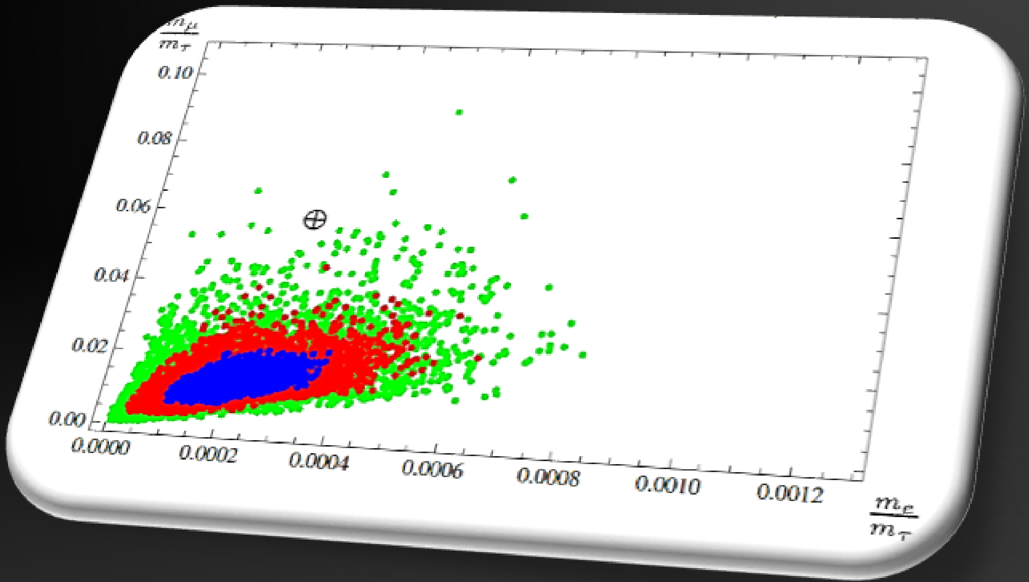

PHYSICAL REVIEW D **93**, 036009 (2016)

Completing constrained flavor violation: Lepton masses, neutrinos, and leptogenesis

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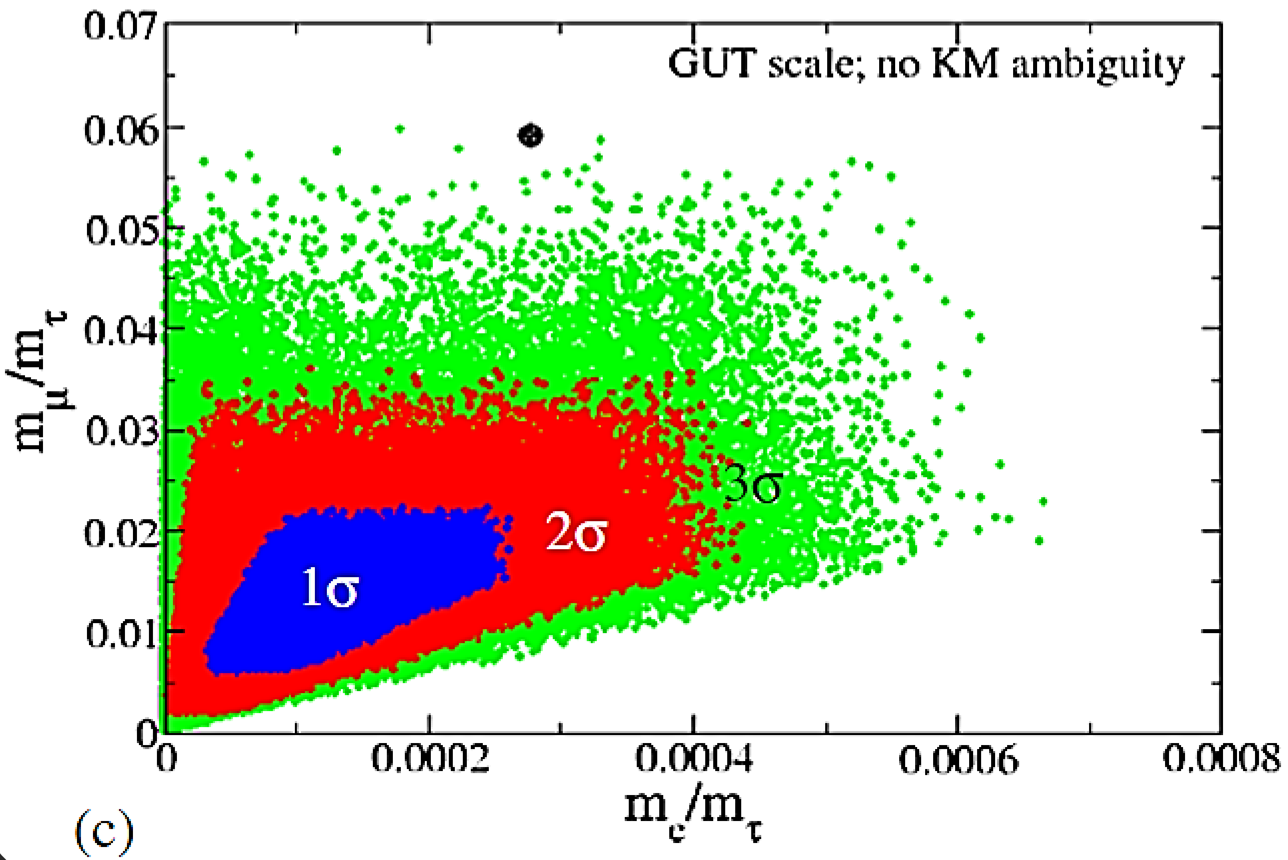
LEPTON MASS RATIOS

- One might question whether prediction is valid at the scale m_Z, whereas the UV flavor physics is expected to come in at a higher scale.
- To assess the effect of going to higher scales, we take advantage of the running Yukawa couplings (represented as running masses) to also test at the GUT scale, taken to be $2 \times 10^{16}\ GeV$.
- For consistency, one also needs the CKM parameters at this scale.


GUT scale; no KM ambiguity
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
m_μ/m_τ
0
0.0002
0.0004
0.0006
0.0008
m_e/m_τ
1σ
2σ
3σ
(c)

LEPTON MASS RATIOS

- In an attempt to address the shortfall in m_μ/m_τ, in the previous work they make a parametric estimate $m_\mu/m_\tau \sim \left(\frac{m_b}{m_s}\right)\left(\frac{m_u}{m_c}\right)\lambda$ (where $\lambda = \cos\theta_C$ is the Wolfenstein parameter), suggesting that a larger value of m_u/m_s could ameliorate this problem.
- According to lattice determinations of the light quark masses, there is no latitude, beyond the usual error estimates, for increasing m_u/m_s.

LEPTON MASS RATIOS

- However, there are still no direct lattice determinations of this ratio.
- Instead, the up and down quarks are always represented by the same field, having a mass of $m_{ud} = (m_u + m_d)/2$.
- Phenomenological input using chiral perturbation theory ($ChPT$) is required to estimate the isospin breaking effects from m_u/m_d.

LEPTON MASS RATIOS

- KM pointed out that at second order in $ChPT$ there is an operator $(\det M)tr(M^{-1}\Sigma)$ that effectively transforms the quark masses by:

$$mu \rightarrow m_u + \alpha m_d m_s$$
$$m_d \rightarrow m_d + \alpha m_u m_s$$
$$m_S \rightarrow m_s + \alpha m_u m_d$$

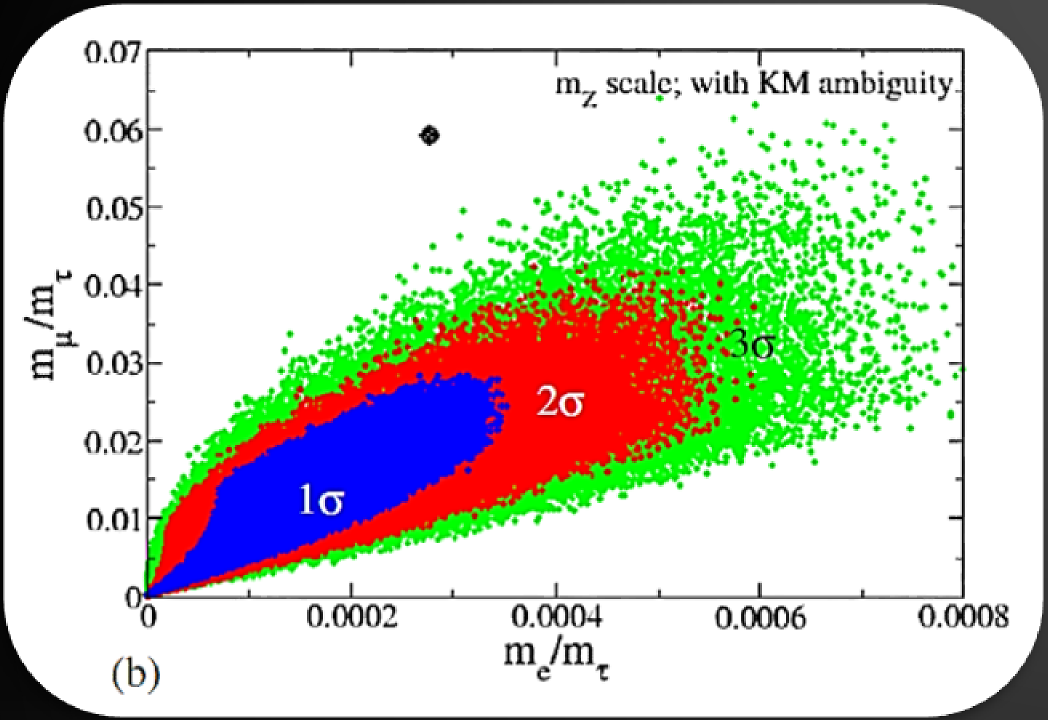
- Where α is a parameter of order $1/\Lambda_{QCD}$.

LEPTON MASS RATIOS

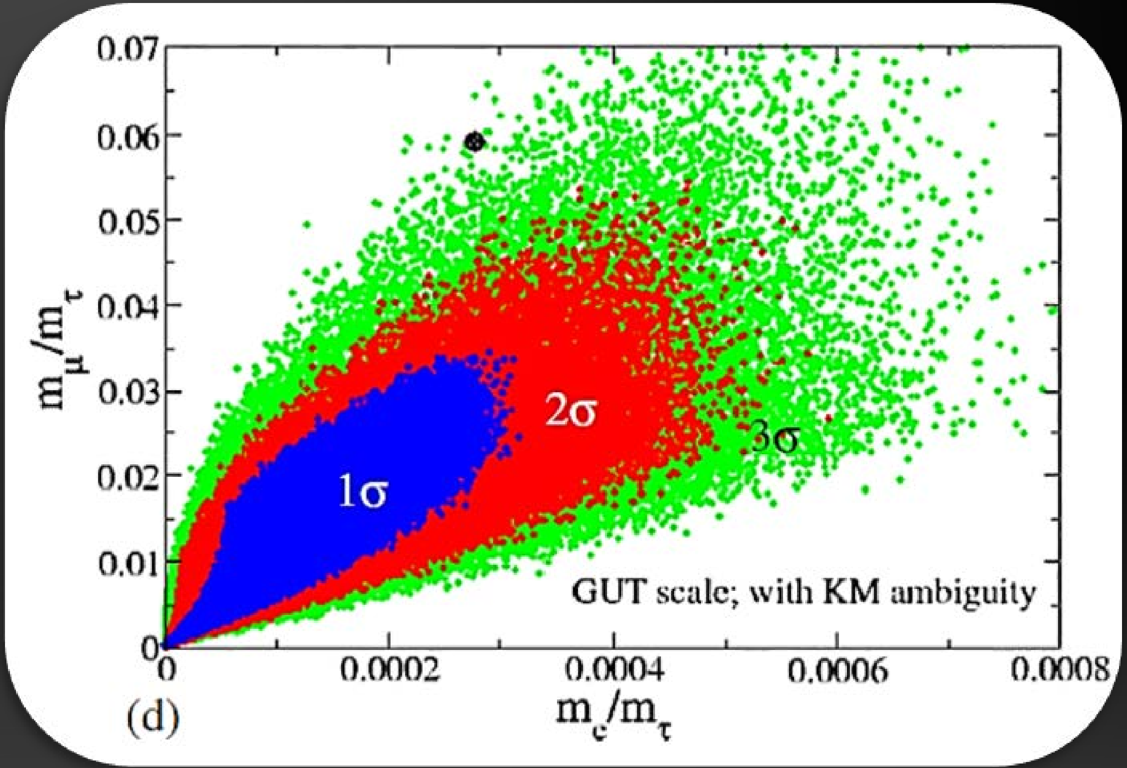
- This could shift the apparent quark masses as deduced from $ChPT$ away from the true values, with a much bigger effect on m_u and m_d than on m_s.
- In principle, α can be determined by comparing enough measured quantities to their second order $ChPT$ predictions (thus determining all the second order coefficients), and this procedure would thus resolve the KM ambiguity.
- On this basis, the isospin breaking effects are considered to be well understood and the ratio $r \equiv m_u/m_d$ is known to high precision $0.46 \pm 0.02 \pm 0.02$ from simulations with 2 ± 1 flavors (2 denoting degenerate u and d, and 1 denoting s).
- However this requires the implicit assumption (not usually stated) that third order $ChPT$ contributions are negligible.

LEPTON MASS RATIOS

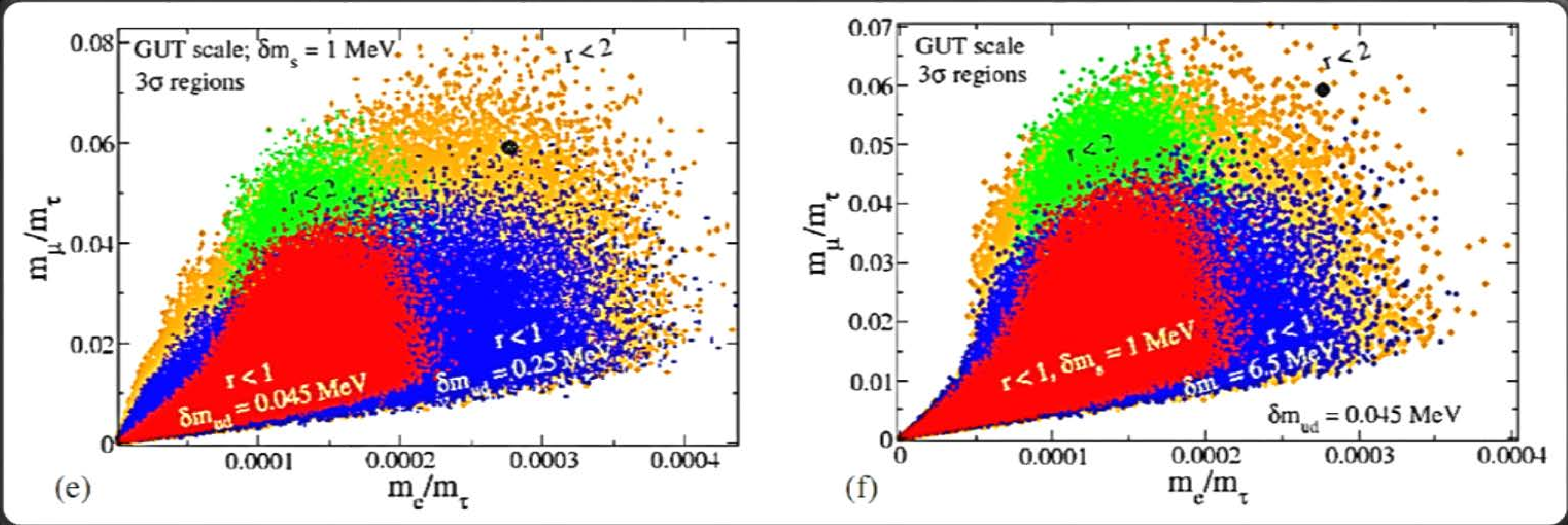
- This assumption can only be rigorously tested by doing a full $1 + 1 + 1$ lattice simulation, leaving room for some doubt about the true value of r.
- Hence we define $r \equiv m_u/m_d$ and allow it to vary away from its standard value, while keeping the errors on m_{ud} and m_s consistent.



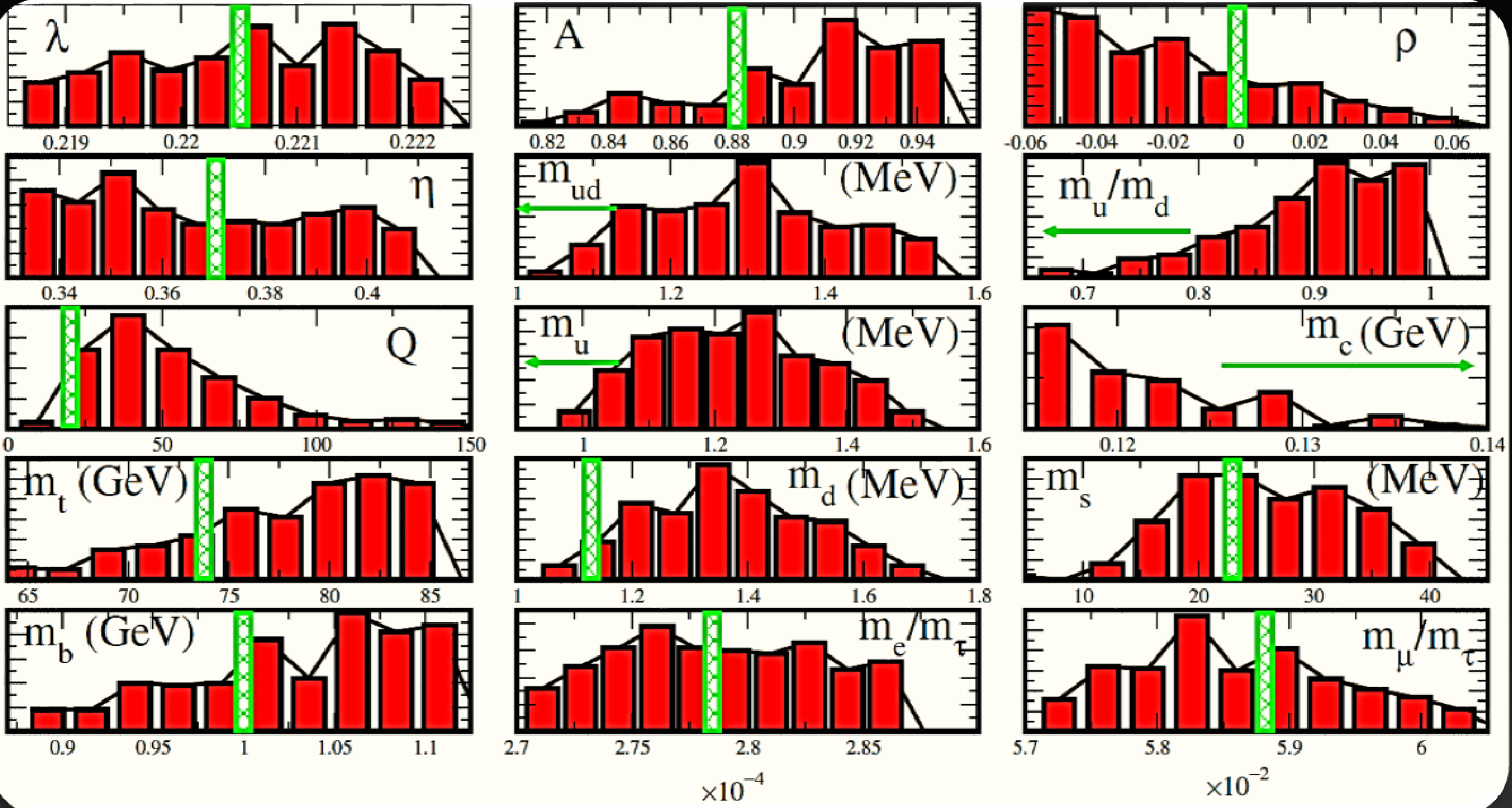
m_Z scale; with KM ambiguity
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
m_μ/m_τ
1σ
2σ
3σ
0
0.0002
0.0004
0.0006
0.0008
m_e/m_τ
(b)



0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
m_μ/m_τ
1σ
2σ
3σ
GUT scale; with KM ambiguity
0
0.0002
0.0004
0.0006
0.0008
m_e/m_τ
(d)



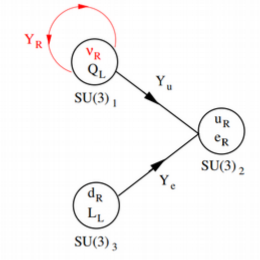
GUT scale; δm_s = 1 MeV
3σ regions
r < 2
r < 2
r < 1
δm_ud = 0.045 MeV
r < 1
δm_ud = 0.25 MeV
m_μ/m_τ
m_e/m_τ
0.08
0.06
0.04
0.02
0
0.0001
0.0002
0.0003
0.0004
(e)
GUT scale
3σ regions
r < 2
r < 2
r < 1, δm_s = 1 MeV
r < 1
δm_s = 6.5 MeV
δm_ud = 0.045 MeV
m_μ/m_τ
m_e/m_τ
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
0.0001
0.0002
0.0003
0.0004
(f)



λ
0.219 0.22 0.221 0.222
A
0.82 0.84 0.86 0.88 0.9 0.92 0.94
ρ
-0.06 -0.04 -0.02 0 0.02 0.04 0.06
η
0.34 0.36 0.38 0.4
m_{ud} (MeV)
1 1.2 1.4 1.6
m_u/m_d
0.7 0.8 0.9 1
Q
0 50 100 150
m_u (MeV)
1 1.2 1.4 1.6
m_c (GeV)
0.12 0.13 0.14
m_t (GeV)
65 70 75 80 85
m_d (MeV)
1 1.2 1.4 1.6 1.8
m_s (MeV)
10 20 30 40
m_b (GeV)
0.9 0.95 1 1.05 1.1
m_e/m_τ
2.7 2.75 2.8 2.85
$\times 10^{-4}$
m_μ/m_τ
5.7 5.8 5.9 6
$\times 10^{-2}$

COMPLETING CFV WITH NEUTRINOS

- To incorporate neutrinos into CFV, one needs to make some assumption about how the right-hand neutrinos ν_R transform under the flavor symmetries.
- In CFV, two species transform non-trivially under each one of the $SU(3)$ subgroups, with the exception of $SU(3)_1$



COMPLETING CFV WITH NEUTRINOS

- To predict the neutrino masses and mixings, we must introduce an additional symmetric spurious field Y_R for the right-handed Majorana neutrino mass matrix, that transforms in the $\bar{6}$ (the symmetric part of $\bar{3} \times \bar{3}$) representation of $SU(3)_1$.
- The Lagrangian invariant under flavor symmetry includes following Yukawa interactions and the right-handed Majorana neutrino mass term

$$\begin{aligned}\mathcal{L} = \quad - & \quad H\bar{Q}_L Y_d d_R - \tilde{H}\bar{Q}_L Y_u u_R - H\bar{L}_L Y_e e_R - \tilde{H}\bar{L}_L Y_\nu \nu_R \\ - & \quad \frac{1}{2} v_R \bar{\nu}_R^c Y_R \nu_R + H.c.\end{aligned}$$

where v_R is a large mass scale and $v_R Y_R$ gives the Majorana mass matrix.

COMPLETING CFV WITH NEUTRINOS

- The flavor symmetries imply that $Y_\nu = \eta' Y_e Y_u^\dagger$.
- After integrating out the heavy neutrino, the light neutrino mass term is

$$\frac{v^2}{2v_R}\bar{\nu}_L Y_\nu Y_R^{-1} Y_\nu^T \nu_L^c + H.c.$$

- Let $\nu_L = L_\nu \nu_m$ denote the relation between the weak eigenstates ν_L and the mass eigenstates ν_m.
  - If Y_e was already diagonal, then L_ν would coincide with the Pontecorvo-Maki-Nakagawa-Sakata ($PMNS$) matrix.
  - However in a basis where Y_e is not diagonal, this is not the case.

COMPLETING CFV WITH NEUTRINOS

- Suppose that the mass term $(v/\sqrt{2})\bar{e}_L Y_e e_R$ is diagonalized by taking $e_L \to L_e e_L$, $e_R \to R_e e_R$, then,

$$U_{PMNS} \equiv U = L_e^\dagger L_\nu$$

where L_ν diagonalizes the neutrino mass matrix via

$$m_\nu = \frac{v^2}{v_R} L_\nu^\dagger Y_\nu Y_R^{-1} Y_\nu^T L_\nu^* = \frac{\eta'^2 v^2}{\eta^2 v_R} L_\nu^\dagger Y_d Y_R^{-1} Y_d^* L_\nu^*$$

Heavy neutrino mass spectra

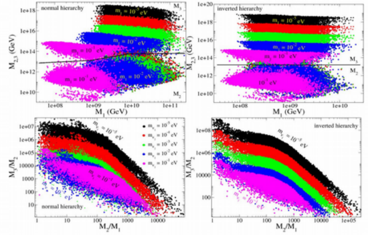
- An interesting feature of the above scenario is that even if L_ν is close to the identity matrix, the factor $L_e^\dagger$ generates large mixing angles in U_{PMNS}.
- This means that the Yukawa matrix Y_R for the sterile neutrino Majorana masses can be very hierarchical despite the large neutrino mixing angles.
- The Majorana matrix Y_R is constrained only by the experimental values of the neutrino masses and mixing angles. Solving for Y_R

$$Y_R = \frac{\eta'^2 v^2}{\eta^2 v_R} (Y_d L_e U)^* m_\nu^{-1} (Y_d L_e U)^\dagger$$

where m_ν is diagonal and L_e is a unitary transformation such that $L_e^\dagger (Y_e Y_e^\dagger) L_e$ is diagonal.

Heavy neutrino mass spectra

- Y_R is a function of the down-to-up quark mass ratios, the *CKM* parameters, the light neutrino masses, the *PMNS* parameters, and four additional phases, as well as the overall scaling factor.
- This requires a choice of the lightest neutrino mass m_{ν_1}, as well as whether the neutrino mass hierarchy is normal or inverted.
- The magnitudes of the heavy neutrino masses M_i are then fixed: $M_i = Y_i v_R$



Conclusions

- Evidence for a simple constraint $Y_d \sim Y_u Y_e^\dagger$ between the fermion Yukawa matrices of the standard model would be very interesting, for gaining insights into the origin of flavor.
- The biggest challenge for this hypothesis is that charged lepton masses are known extremely well. We have shown that if the prediction actually applies at a high scale such as the GUT scale, and if the up-to-down quark mass ratio is somewhat larger at this scale than at low energies, the problem with lepton masses can be overcome. It is conceivable that the effects of some scalar associated with flavor violation affects the running of the Yukawa couplings in such a way, as the renormalization group scale crosses its mass threshold.

Conclusions

- One of the hints that the $Y_d \sim Y_u Y_e^\dagger$ relation might be correct is that it naturally leads to large mixing angles in the leptonic sector.
- We have suggested a completion of the framework that includes the neutrino Yukawa matrix, such that $Y_\nu \sim Y_e Y_u^\dagger$. This is not as predictive as the original relation, because it does not specify the structure of the heavy neutrino mass matrix. The latter we have fixed (up to phases) using experimental constraints on neutrino masses and mixings.

GRACIAS