Modern Aspects of Perturbative QFT and Gravity

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Outline

- Introduction and Motivation,
- The Spinor Helicity Formalism, Little Group Scaling,
- Recursion Relations,
- SHF for massive particles,
- Final Comments.

Motivation

"Scattering experiments are crucial to understand the fundamental blocks in nature".

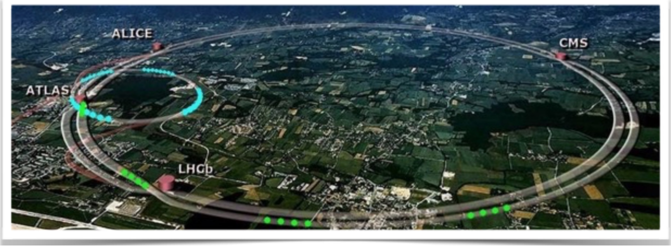
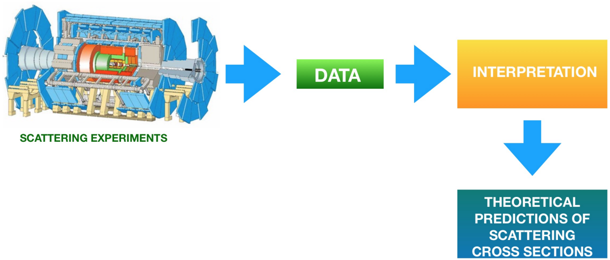


Figure: CERN (Geneve, Switzerland)

Motivation

The Standard Model of elementary particles was developed largely because scattering experiments, (the discovery the gauge bosons $W^{\pm}$ y Z^{0} and the gluons and quarks and more recently the Higgs bosons).



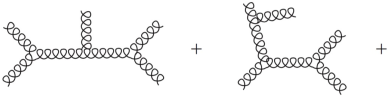
Motivation

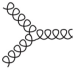
It is well know (theoretically and experimentally) that **QFT** describes the elementary particles and the fundamental forces of nature.
The DCS (calculated with **QFT**) that *connect* theory with experiments is proportional to the modulus squared of the amplitude.

$$\boxed{\frac{d\sigma}{d\Omega} \propto |\mathcal{A}|^2}$$

Pure YM theory

If we consider a pure Yang - Mills theory, computing the scattering amplitude (modulus square) at three level i.e. for 5 gluons, we have:





$$\rightarrow g f^{abc}[(p_1 - p_2)_\rho \eta_{\mu\nu} + (p_2 - p_3)_\mu \eta_{\nu\rho} + (p_3 - p_1)_\nu \eta_{\rho\mu}]$$

After brute force computing, part of the modulus square SA (25 Feynman diagrams) is shown

$k_1 \cdot k_4 \, \varepsilon_2 \cdot k_1 \, \varepsilon_1 \cdot \varepsilon_3 \, \varepsilon_4 \cdot \varepsilon_5$

But using modern techniques, we obtain a very simple and compact expression for the 5 gluons HA

$$A_5(1^{\pm},2^{+},3^{+},4^{+},5^{+}) = 0$$

$$A_5(1^{-},2^{-},3^{+},4^{+},5^{+}) = i\frac{\langle 1\,2\rangle^4}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,\langle 5\,1\rangle}$$

Spinor Helicity Formalism (a pragmatic point of view)

The SHF is based in the following observation:

Fields with spin-1 transform in the $(\frac{1}{2}, \frac{1}{2})$ representation of the Lorentz group.

So we are able to express the 4-momentum of any particle as a biespinor: $p_\mu \to p_{a\dot{a}}$

$$\boxed{p_{a\dot{a}} = p_\mu \sigma^\mu_{a\dot{a}}} \quad (1)$$

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}$$

$$\sigma^\mu_{a\dot{a}} = (I, \vec{\sigma}),\ \bar{\sigma}^{\mu\dot{a}a} = (I, -\vec{\sigma})$$

4-component spinors

4-Component Dirac spinors ($u_s(\vec{p})$ and $v_s(\vec{p})$) are solutions of the following EOMs:

$$\boxed{(\not{p} + m)u_s(\vec{p}) = 0,} \tag{2}$$

$$\boxed{(-\not{p} + m)v_s(\vec{p}) = 0,} \tag{3}$$

with $s = \pm$.

Massless Case (*textbooks*)

For massless Dirac spinors we know that the following equation exist: $u_s(\vec{p})\bar{u}_s(\vec{p}) = \frac{1}{2}(1+s\gamma_5)(-\not{p})$, for instance if $s = -$, we have:

$$u_-(\vec{p})\bar{u}_-(\vec{p}) = \frac{1}{2}(1-\gamma_5)(-\not{p}) = \begin{pmatrix} 0 & -p_{a\dot{a}} \\ 0 & 0 \end{pmatrix}$$

where

$$u_-(\vec{p}) = \begin{pmatrix} \phi_a \\ 0 \end{pmatrix}$$

ϕ_a is a 2-component numerical spinor,

also $\bar{u}_-(\vec{p}) = (0, \phi^*_{\dot{a}})$, all this lead to $\boxed{p_{a\dot{a}} = -\phi_a\phi^*_{\dot{a}}}$

The key of the SHF is to considerate ϕ_a as the fundamental object and express the momenta of the particles in terms of ϕ_a.

Highly convenient and powerful notation

If p and k are the momenta and ϕ_a, κ_a their associated spinors, we can define the following products of spinors:

$$\boxed{[pk] = \phi^a \kappa_a = \bar{u}_+(\vec{p})u_-(\vec{k}) = -[kp]} \tag{4}$$

$$\boxed{\langle pk \rangle = \phi^*_{\dot{a}} \kappa^{*\dot{a}} = \bar{u}_-(\vec{p})u_+(\vec{k}) = -\langle kp \rangle} \tag{5}$$

Notation for 4-components Dirac Spinor

$$|p] = u_-(\mathbf{p}) = v_+(\mathbf{p}) \; ,$$

$$|p\rangle = u_+(\mathbf{p}) = v_-(\mathbf{p}) \; ,$$

$$[p| = \overline{u}_+(\mathbf{p}) = \overline{v}_-(\mathbf{p}) \; ,$$

$$\langle p| = \overline{u}_-(\mathbf{p}) = \overline{v}_+(\mathbf{p}) \; .$$

where spinor products as $\bar{u}_s(\vec{p})\, u_s(\vec{k}) = 0 \;\forall\; s = \pm$, or more nice, we have:

$$\boxed{\langle pk] = [pk\rangle = 0} \tag{6}$$

Polarizations 4-vectores

If we want to apply the SHF to QED, we need to write polarization vectors in term of 2-component numerical spinors.

$$\varepsilon_+^\mu(k) = -\frac{\langle q|\gamma^\mu|k]}{\sqrt{2}\,\langle q\,k\rangle}\,,$$

$$\varepsilon_-^\mu(k) = -\frac{[q|\gamma^\mu|k\rangle}{\sqrt{2}\,[q\,k]}\,,$$

where they satisfy $p \cdot \mathcal{E}_s = 0$ and $\mathcal{E}_s^2 = 1\ \forall\ s = \pm$.

- **The contraction with γ matrix is as follows: :**

$$\not{\epsilon}_+(k,q) = \frac{1}{\sqrt{2}\langle qk\rangle}\langle q|\gamma^\mu|k]\gamma_\mu = \frac{2}{\sqrt{2}}(|k]\langle q| + |q\rangle[k|)$$

$$= \frac{\sqrt{2}}{\langle qk\rangle}(|k]\langle q| + |q\rangle[k|), \tag{7}$$

$$\not{\epsilon}_-(k,q) = \frac{1}{\sqrt{2}[qk]}[q|\gamma^\mu|k\rangle\gamma_\mu = \frac{1}{\sqrt{2}[qk]}\langle k|\gamma^\mu|q]\gamma_\mu$$

$$= \frac{\sqrt{2}}{[qk]}(|k]\langle q| + |q\rangle[k|). \tag{8}$$

Some of the most important formulas that are needed to compute scattering amplitudes :

$$[ij] = -[ji],$$

$$\langle ij\rangle = [ji]^*,$$

$$\langle ij\rangle[ji] = \langle ij\rangle\langle ij\rangle^* = |\langle ij\rangle|^2,$$

$$\langle ij\rangle[ji] = -2k_i\cdot k_j = s_{ij},$$

$$\langle i|\gamma_\mu|j] = [j|\gamma_\mu|i\rangle,$$

$$\langle i|\gamma_\mu|j]\langle k|\gamma^\mu|l] = 2\langle ik\rangle[lj],$$

$$\langle ab\rangle\langle cd\rangle = \langle ac\rangle\langle bd\rangle + \langle ad\rangle\langle cb\rangle,$$

$$\sum_{k=1}^{n}\langle ik\rangle[kj] = 0,$$

Let us compute the 5 points scattering amplitude for the following process $e^+e^- \to q\bar{q}g$.

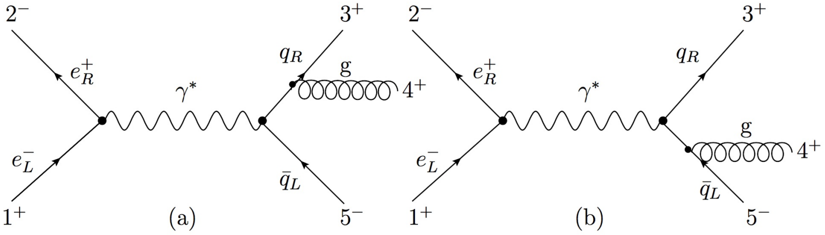


Figura 4: Diagramas contribuyentes para el proceso $e^+e^- \to q\bar{q}g$

We will use the color-ordered amplitude.

$e^+e^- \to q\bar{q}g$

$$\mathcal{M}_5 = (-ie)^2(2\sqrt{2})(-ig)Q_{\bar{q}}Q_q(T^{\mathcal{A}_4})_{i_3}^{\bar{j}_5}\mathcal{A}_5(1_{\bar{e}}^+, 2_e^-, 3_q^+, 4_g^+, 5_{\bar{q}}^-). \quad (9)$$

Building the kinematical factor (partial amplitude) still "a la Feynman", we found

$$\mathcal{A}_5(1_{\bar{e}}^+, 2_e^-, 3_q^+, 4_g^+, 5_{\bar{q}}^-) = \frac{1}{2\sqrt{2}}\langle 2|\gamma^\mu|1]\frac{1}{s_{12}}\Bigg([3|\not{\epsilon}_+(4,q)\frac{(\not{3}+\not{4})}{s_{34}}\gamma_\mu|5\rangle$$
$$+ [3|\gamma_\mu\frac{(\not{4}+\not{5})}{s_{45}}\not{\epsilon}_+(4,q)|5\rangle\Bigg). \quad (10)$$

One of the several advantages of the SHF, is that one can compute by blocks, i.e. $[3|\not\epsilon_+(4,q)$ and $\not\epsilon_+(4,q)|5\rangle$,

$$[3|\not\epsilon_+(4,q) = \frac{\sqrt{2}}{\langle q4\rangle}[3|(|4]\langle q| + |q\rangle[4|) \tag{11}$$

$$= \frac{\sqrt{2}}{\langle q4\rangle}([34]\langle q| + \cancelto{0}{[3q\rangle}[4|) = \frac{\sqrt{2}}{\langle q4\rangle}[34]\langle q|, \tag{12}$$

$$\not\epsilon_+(4,q)|5\rangle = \frac{\sqrt{2}}{\langle q4\rangle}(|4]\langle q| + |q\rangle[4|)|5\rangle \tag{13}$$

$$= \frac{\sqrt{2}}{\langle q4\rangle}(|4]\langle q5\rangle + |q\rangle\cancelto{0}{[45\rangle}) = \frac{\sqrt{2}}{\langle q4\rangle}|4]\langle q5\rangle. \tag{14}$$

After some algebra, one can found that:

$$\mathcal{A}_5 = \frac{2[34]}{2s_{12}s_{34}\langle 54\rangle}\left(\langle 53\rangle[31]\langle 25\rangle + \langle 54\rangle[41]\langle 25\rangle\right) \tag{15}$$

$$= \frac{[34]\langle 25\rangle}{s_{12}s_{34}\langle 54\rangle}\left(\langle 53\rangle[31] + \langle 54\rangle[41]\right) \tag{16}$$

$$= \frac{[34]\langle 25\rangle}{s_{12}s_{34}\langle 54\rangle}\left(-[12]\langle 25\rangle - [14]\langle 45\rangle + [14]\langle 45\rangle(-1)^2\right) \tag{17}$$

$$= -\frac{[34]\langle 25\rangle[12]\langle 25\rangle}{\langle 12\rangle[21]\langle 34\rangle[43]\langle 54\rangle} = -\frac{(-1)\cancel{[34]}\langle 25\rangle^2(-1)\cancel{[12]}}{\langle 12\rangle\cancel{[21]}\langle 34\rangle\cancel{[43]}\langle 54\rangle} \tag{18}$$

$$= -\frac{(-1)\cancel{[34]}\langle 25\rangle^2(-1)\cancel{[12]}}{\langle 12\rangle\cancel{[21]}\langle 34\rangle\cancel{[43]}(-1)\langle 45\rangle} = \frac{\langle 25\rangle^2}{\langle 12\rangle\langle 34\rangle\langle 45\rangle}. \tag{19}$$

We have found the partial amplitude for the process $e^+e^- \to q\bar{q}g$. It can be notices that the final expression is a very simple and nice expression.

$$\boxed{\mathcal{A}_5 = \frac{\langle 25\rangle^2}{\langle 12\rangle\langle 34\rangle\langle 45\rangle},}$$

¿Is it possible to make this kind of calculations even simpler?

Little Group Scaling

The two components Weyl spinors are invariant under the scaling

$$\boxed{|p\rangle \to t\,|p\rangle,}$$

$$\boxed{|p] \to t^{-1}\,|p]}$$

this is called little group scaling.

Little Group

The LG is the group of transformations that leave the momentum of an on-shell particle invariant.

What an amplitude is made of?

The Feynman diagram consists of:

- Propagators.
- Vertices.
- External line rules.

When only massless particles are involved, the amplitude can always be rewritten in terms of angle and square brackets. *But only the external line rules scale under little group*.

Under little group scaling of each particle $i = 1, 2, \ldots, n$, the on-shell amplitude transform homogeneously with weight $-2h_i$, where h_i is the helicity of particle i.

$$\mathcal{A}_n(\{|1\rangle, |1], h_1\}, \ldots, \{t_i|i\rangle, t_i^{-1}|i], h_i\}, \ldots) = t_i^{-2h_i}\mathcal{A}_n(\ldots\{|i\rangle, |i], h_i\}\ldots) \tag{20}$$

3-point amplitudes

$$\mathcal{A}_3(1^{h_1}, 2^{h_2}, 3^{h_3}) = c\langle 12\rangle^{h_3-h_1-h_2}\langle 13\rangle^{h_2-h_1-h_3}\langle 23\rangle^{h_1-h_2-h_3}$$

The helicity structure uniquely fixes the 3-particle amplitude up to an overall constant.

Considering $\mathcal{A}_3(g_1^-, g_2^-, g_3^+)$, $h_1 = -1$, $h_2 = -1$ and $h_3 = 1$. Using the last result, we found the following expression for the 3 gluons amplitude

$$\boxed{\mathcal{A}_3(g_1^-, g_2^-, g_3^+) = g\frac{\langle 12\rangle^3}{\langle 13\rangle\langle 23\rangle} = -g\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 31\rangle}}$$

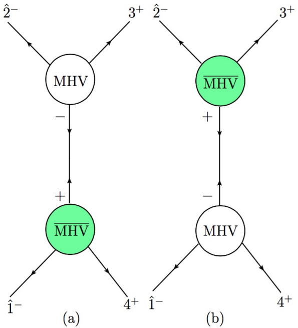
This matches the calculations using the SHF, but this is much simpler than the direct Feynman diagram calculation.

Recursivity

Tree amplitudes are rational functions of the kinematic invariants, and understanding their pole structure is key to the derivation of on-shell recursive methods that provide a very efficient alternative to Feynman diagrams. These recursive methods allow one to construct on-shell n-particle tree amplitudes from input of on-shell amplitudes with fewer particles.

BCFW

Four particles amplitudes $\mathcal{A}^{arbol}_{4(a,b)}$



$$\boxed{\mathcal{A}_n = \sum_{diagramas\, I} \hat{\mathcal{A}}_L(z_I)\frac{1}{\hat{P}_I^2}\hat{\mathcal{A}}_R(z_I),} \tag{21}$$

$$\boxed{\mathcal{A}_{4(a)}^{arbol} = -\frac{1}{z_{23}}\mathcal{A}_3(\hat{1}^-, \hat{k}_{23}^+, 4^+)\left(\frac{i}{[32]\langle 13\rangle}\right)\mathcal{A}_3(\hat{2}^-, 3^+, -\hat{k}_{23}^-) = 0,}$$

$$\boxed{\mathcal{A}_{4(b)}^{arbol} = \frac{i\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 43\rangle\langle 41\rangle},}$$

Gravitons Amplitude

From String Theory we know the amazing result that relates scattering amplitudes of open strings with scattering amplitudes of close strings.

KLT

$$\boxed{\mathcal{A}_4(\text{Gravity}) = \mathcal{A}_4(\text{Yang-Mills}) \times \mathcal{A}_4(\text{Yang-Mills})}$$

This recursion relation works just at tree level amplitude.
Now, we know that there exist a more general duality BCJ.

To compute the 4 gravitons amplitude $\mathcal{A}_4(1,2,3,4)$

- Use the known result for 3 point amplitudes in YM (Using SHF or little group scaling).
- Use BCFW to compute the 4 point amplitude in YM.
- Finally use KLT RR.

The final result

$$\boxed{\mathcal{A}_4(gravitons) = \left(\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}\right)^2 ,} \tag{22}$$

$$\boxed{\overline{\mathcal{A}}_4(gravitons) = \left(\frac{[12]^4}{[12][23][34][41]}\right)^2 .} \tag{23}$$

What about massive particles ?

All the massive spinors are as follows

$$
\begin{aligned}
u_- &= |r] + \frac{m}{\langle rq\rangle}|q\rangle \quad , \quad u_+ = \frac{m}{[rq]}|q] + |r\rangle; \\
v_+ &= |r] - \frac{m}{\langle rq\rangle}|q\rangle \quad , \quad v_- = -\frac{m}{[rq]}|q] + |r\rangle; \\
\bar{u}_- &= \frac{m}{[qr]}[q| + \langle r| \quad , \quad \bar{u}_+ = [r| + \frac{m}{\langle qr\rangle}\langle q|; \\
\bar{v}_+ &= -\frac{m}{[qr]}[q| + \langle r| \quad , \quad \bar{v}_- = [r| - \frac{m}{\langle qr\rangle}\langle q|.
\end{aligned}
$$

In order to apply the SHF (massive or massless) to the standard model of particle physics, we just need to have the massive polarization vector in term of 2-component numerical spinors.

We already know the massive polarization vector in terms of spinors.

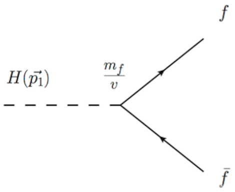
$$\epsilon_+^\mu = \frac{\langle q|\gamma^\mu|r]}{\sqrt{2}\langle rq\rangle},$$
$$\epsilon_-^\mu = \frac{\langle r|\gamma^\mu|q]}{\sqrt{2}[qr]},$$

$$\begin{aligned}\epsilon_0^\mu &= \frac{1}{2m}\left(\langle r|\gamma^\mu|r] - \alpha\langle q|\gamma^\mu|q]\right)\\ &= \frac{1}{m}r^\mu + \frac{m}{2p\cdot q}q^\mu.\end{aligned}$$

Let us see some examples.

2-body Higgs decay $h(p_1) \to f(p_2)\bar{f}(p_3)$

From the Feynman diagram we get the helicity amplitudes (HAs) (as is usual)



$$\boxed{\mathcal{M}_{\lambda_2\lambda_3}(p_1,p_2,p_3) = \frac{m_f}{v}\bar{u}_{\lambda_2}(p_2)v_{\lambda_3}(p_3),} \tag{24}$$

$\lambda_2\lambda_3$	$\mathcal{M}_{\lambda_2\lambda_3}$
$--$	$\frac{m_f}{v[q_2q_3]}(s_{q_2q_3} - m_f^2)$
$++$	$\frac{m_f}{v\langle q_2q_3\rangle}(s_{q_2q_3} - m_f^2)$

Finally the averaged squared amplitude is then:

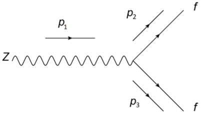
$$\boxed{\langle|\mathcal{M}|^2\rangle = 2|\mathcal{M}_{--}|^2 = \frac{2m_f^2}{v^2 s_{q_2q_3}}(s_{q_2q_3} - m_f^2)^2 = \frac{y^2}{v^2}(1-4y^2),} \tag{25}$$

with $y = \frac{m_f}{M_h}$. Then the decay width Γ goes as follows

$$\boxed{\Gamma(h \to f\bar{f}) = \frac{\alpha_W M_h y^2}{8}(1-4y^2)^{3/2}.} \tag{26}$$

2-body Z boson decay $Z(p_1) \to f(p_2)\bar{f}(p_3)$

Again from the Feynman diagram we get the helicity amplitudes



$$\boxed{\mathcal{M} = \frac{1}{2} g_Z \epsilon_\mu(p_1)\bar{u}(p_2)\gamma^\mu(v_f - a_f\gamma^5)v(p_3)} \tag{27}$$

$\lambda_1\lambda_2\lambda_3$	$\mathcal{M}_{\lambda_1\lambda_2\lambda_3}$
$++-$	$\frac{1}{2}g_Z\frac{\langle p_2\vert\gamma_\mu\vert r_1]}{\sqrt{2}\langle r_1p_2\rangle}(v_f-a_f)[p_2\vert\gamma^\mu\vert p_3\rangle = \frac{g_Z(v_f-a_f)}{\sqrt{2}}\frac{\langle p_2p_3\rangle[r_1p_2]}{\langle r_1p_2\rangle}$
$0+-$	$\frac{1}{2}g_Z\left(\frac{1}{M}r_{1\mu}+\frac{M}{2p_{12}}p_{2\mu}\right)(v_f-a_f)[p_2\vert\gamma^\mu\vert p_3\rangle = \frac{g_Z(v_f-a_f)}{\sqrt{2}M}\langle r_1p_3\rangle[r_1p_2]=0$
$-+-$	$\frac{1}{2}g_Z\frac{\langle r_1\vert\gamma_\mu\vert p_2]}{\sqrt{2}[p_2r_1]}(v_f-a_f)[p_2\vert\gamma^\mu\vert p_3\rangle = 0$

Finally the squared averaged scattering amplitude is as follows

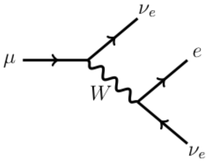
$$\langle|\mathcal{M}|^2\rangle = \frac{1}{3}\left(|\mathcal{M}_{++-}|^2 + |\mathcal{M}_{--+}|^2\right) = \frac{g_Z^2 M^2}{3}\left(|v_f|^2 + |a_f|^2\right) \tag{28}$$

The decay width for this channel is then:

$$\Gamma\left(Z \to f\bar{f}\right) = \frac{g_Z^2 M}{48\pi}\left(|v_f|^2 + |a_f|^2\right). \tag{29}$$

3-body Muon Decay $\mu(p_1) \to \bar{\nu}_{e^-}(p_2)\,\nu_\mu(p_3)\,e^-(p_4)$

From the Feynman diagram we get the helicity amplitudes (HAs) (as is usual)



$$\mathcal{M}_{\lambda_1\lambda_2\lambda_3\lambda_4} = \left(\frac{g_W}{\sqrt{8}M_W}\right)^2 [\bar{u}_{\lambda_3}(p_3)\gamma^\mu(1-\gamma_5)u_{\lambda_1}(p_1)]\,[\bar{u}_{\lambda_4}(p_4)\gamma_\mu(1-\gamma_5)v_{\lambda_2}(p_2)]\,,$$
$$= \left(\frac{g_W}{\sqrt{8}M_W}\right)^2 \mathcal{A}^\mu_{\lambda_3\lambda_1}\mathcal{B}_{\mu\lambda_4\lambda_2},$$

$\lambda_1\lambda_2\lambda_3\lambda_4$	$\mathcal{A}^{\mu\lambda_3\lambda_1}$	$\mathcal{B}_\mu^{\lambda_4\lambda_2}$	$\mathcal{A}^{\mu\lambda_3\lambda_1}\mathcal{B}_\mu^{\lambda_4\lambda_2}$	$\mathcal{M}\quad(p_2=q_1,\, p_3=q_4)$
$+-+-$	$2[p_3\vert\gamma^\mu\vert r_1\rangle$	$2[r_4\vert\gamma_\mu\vert p_2\rangle$	$4\langle p_2 r_1\rangle\langle p_3 r_4\rangle$	$\left(\frac{g_W}{\sqrt{2}m_\mu}\right)^2\langle p_2 r_1\rangle[p_3 r_4]$
$+++-$	$2[p_3\vert\gamma^\mu\vert r_1\rangle$	$\frac{2m_e}{[q_4 r_4]}[q_4\vert\gamma_\mu\vert p_2\rangle$	$4\frac{m_e}{[q_4 r_4]}\langle p_2 r_1\rangle\langle p_3 q_4\rangle$	0
$--++$	$\frac{2m_\mu}{\langle r_1 q_1\rangle}[p_3\vert\gamma^\mu\vert q_1\rangle$	$2[r_4\vert\gamma_\mu\vert p_2\rangle$	$4\frac{m_\mu}{\langle r_1 q_1\rangle}\langle p_2 q_1\rangle[p_3 r_4]$	0
$-++-$	$\frac{2m_\mu}{\langle r_1 q_1\rangle}[p_3\vert\gamma^\mu\vert q_1\rangle$	$\frac{2m_e}{[q_4 r_4]}[q_4\vert\gamma_\mu\vert p_2\rangle$	$4\frac{m_\mu m_e}{\langle r_1 q_1\rangle[q_4 r_4]}\langle p_2 q_1\rangle[p_3 q_4]$	0

The squared and averaged amplitude for the muon decay is:

$$\boxed{\langle|\mathcal{M}^{+-+-}|^2\rangle = \frac{1}{2}|\mathcal{M}^{+-+-}|^2 = 2\left(\frac{g_W}{M_W}\right)^4 (p_1\cdot p_2)(p_3\cdot p_4)} \tag{30}$$

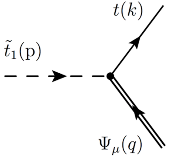
From this result we can arrive to the decay width, which agrees with result of textbooks.

Gravitino wave functions

We will consider a model in a local supersymmetric field theory ($\mathcal{N} = 1$ SUGRA) where the gravitino is the lightest supersymmetric particle (LSP), therefore a good DM candidate.

$$\tilde{\Psi}^{\mu}_{++}(p) = \epsilon^{\mu}_{+}(p) u_{+}(p),$$
$$\tilde{\Psi}^{\mu}_{--}(p) = \epsilon^{\mu}_{-}(p) u_{-}(p)$$
$$\tilde{\Psi}^{\mu}_{+}(p) = \sqrt{\frac{2}{3}}\epsilon^{\mu}_{0}(p) u_{+}(p) + \frac{1}{\sqrt{3}}\epsilon^{\mu}_{+}(p) u_{-}(p),$$
$$\tilde{\Psi}^{\mu}_{-}(p) = \sqrt{\frac{2}{3}}\epsilon^{\mu}_{0}(p) u_{-}(p) + \frac{1}{\sqrt{3}}\epsilon^{\mu}_{-}(p) u_{+}(p).$$

Two body NLSP Stop decay ($\tilde{t} \to t\,\tilde{G}$)



$$\boxed{\mathcal{M}_{\lambda_2\lambda_3} = -\frac{1}{\sqrt{2}M}\bar{\psi}^{\mu}_{\lambda_2}(p_2)\gamma_\alpha\gamma_\mu p_1^\alpha \hat{P}_R u_{\lambda_3}(p_3)} \tag{31}$$

$\lambda_2\lambda_3$	$\mathcal{M}_{\lambda_2\lambda_3}$
$-+$	$-\frac{\langle r_2 q_2\rangle}{\sqrt{3}M m_{\tilde{G}} s_{r_2 q_2}}\left(s^2_{r_2 q_2} - m^2_{\tilde{G}} m^2_t\right)$
$+-$	$-\frac{[r_2 q_2] m_t}{\sqrt{3}M s^2_{r_2 q_2}}\left(s^2_{r_2 q_2} - m^2_{\tilde{G}} m^2_t\right)$

Squaring the helicity amplitudes of the last Table, we obtain the following expression for the total squared amplitude

$$\langle |\mathcal{M}|^2 \rangle = |\mathcal{M}_{-+}|^2 + |\mathcal{M}_{+-}|^2, \tag{32}$$

$$= \frac{\left(s_{r_2q_2}^4 - (m_t m_{\tilde{G}})^4\right)\left(s_{r_2q_2}^2 - (m_t m_{\tilde{G}})^2\right)}{3M^2 m_{\tilde{G}}^2 s_{r_2q_2}^3}, \tag{33}$$

$$= \frac{(m_{\tilde{t}}^2 - m_{\tilde{G}}^2 - m_t^2)\left((m_{\tilde{t}}^2 - m_t^2 - m_{\tilde{G}}^2)^2 - 4m_t^2 m_{\tilde{G}}^2\right)}{3M^2 m_{\tilde{G}}^2}, \tag{34}$$

To appreciate the power and efficiency of the SHF, we can remember the completeness relation for the spin-3/2 Gravitino Field.

$$\sum_{\tilde{\lambda}=1}^{3} \Psi_\mu(\vec{p}_1,\tilde{\lambda})\overline{\Psi}_\nu(\vec{p}_1,\tilde{\lambda}) = -(\not{p}_1 + m_{\tilde{G}}) \times \left\{ \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{m_{\tilde{G}}^2} \right) - \frac{1}{3}\left(g_{\mu\sigma} - \frac{p_\mu p_\sigma}{m_{\tilde{G}}^2} \right)\left(g_{\nu\lambda} - \frac{p_\nu p_\lambda}{m_{\tilde{G}}^2} \right)\gamma^\sigma\gamma^\lambda \right\}$$

And we still need to take into account other fields to compute the trace.

Conclusions

- The scattering amplitudes have a physical relevance. But the amplitudes themselves have a very interesting mathematical structure. Understanding this structure guides us towards more efficient methods to calculate amplitudes.
- We are exploring how to implement more sophisticate methods to our calculation with massive gravitinos, namely **BCJ** and **BCFW** relations. We would like to apply this techniques to some relevant process in modern Cosmology.

Thank you

In spite of the tremendous difficulties lying ahead, I feel that S-matrix theory is far from dead and that . . . much new interesting mathematics will be created by attempting to formalize it.

"Tullio Regge"

Light Cone Decomposition (LCD)

Let $\mathbf{p}^{\mu}$ be any time-like 4-momentum, we can decompose it into 2 light-like four momenta as follows. Let $\mathbf{q}^{\mu}$ be an arbitrary light-like four momentum, and define

$$\boxed{r^{\mu} \equiv p^{\mu} - \alpha q^{\mu}} . \tag{35}$$

We want that r be light-like too, so we impose $r^2 = 0$; then

$$0 = (p^{\mu} - \alpha q^{\mu})(p_{\mu} - \alpha q_{\mu}) = p^2 - 2\alpha p^{\mu} q_{\mu} + \alpha^2 q^2, \tag{36}$$

but $q^2 = 0$, therefore $\alpha = \frac{p^2}{2p \cdot q}$.

Now we can see how LCD applies to massive spinors. Remember Dirac equation:

$$\boxed{(\not{p}+m)u_s(\vec{p})=0}, \tag{37}$$

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$$\boxed{(\not{p} + m)u_s(\vec{p}) = 0}, \tag{37}$$

when one consider the 4-component Dirac spinor in terms of two 2-component spinors

$$u = \begin{pmatrix} \chi_a \\ \xi^{\dot{a}} \end{pmatrix}, \tag{38}$$

Dirac equation is equivalent to the following system

$$p_{a\dot{a}}\xi^{\dot{a}} + m\chi_a = 0, \tag{39}$$

$$p^{\dot{a}a}\chi_a + m\xi^{\dot{a}} = 0. \tag{40}$$

We have the last two equation in such a way that is easy applies LCD, this reads as

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- $-\left(|r]_a\langle r|_{\dot{a}} + \alpha|q]_a\langle q|_{\dot{a}}\right)$

$$p_{a\dot{a}}\xi^{\dot{a}} + m\chi_a = 0 \tag{41}$$

$$p^{\dot{a}a}\chi_a + m\xi^{\dot{a}} = 0 \tag{42}$$

- $-\left(|r\rangle^{\dot{a}}[r|^{a} + \alpha|q\rangle^{\dot{a}}[q|^{a}\right)$

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$$\boxed{\left(|r]_a\langle r|_{\dot{a}} + \alpha|q]_a\langle q|_{\dot{a}}\right)\xi^{\dot{a}} = m\chi_a} \tag{43}$$

$$\boxed{\left(|r\rangle^{\dot{a}}[r|^a + \alpha|q\rangle^{\dot{a}}[q|^a\right)\chi_a = m\xi^{\dot{a}}} \tag{44}$$

The solutions for the last 2 equations are as follows:

$$\boxed{u = \begin{pmatrix} \frac{m}{[rq]}|q]_a \\ |r\rangle^{\dot{a}} \end{pmatrix}} \tag{45}$$

with the spinors $\chi_a = \frac{m}{[rq]}|q]_a$ and $\xi^{\dot{a}} = |r\rangle^{\dot{a}}$.